EFFICIENT NUMERICAL SOLUTION OF THE LAMBERT'S PROBLEM WITH SOLAR SAILING

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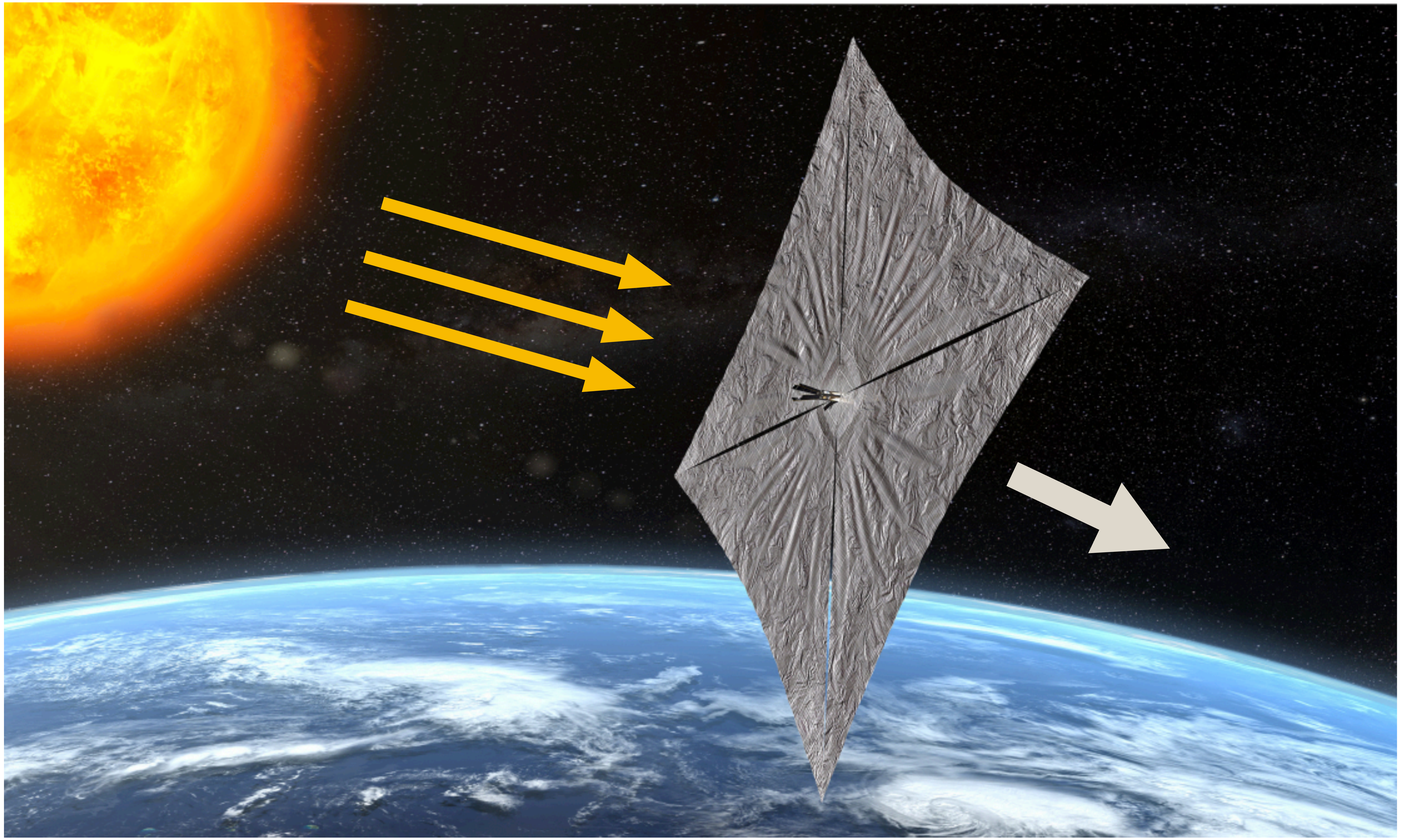
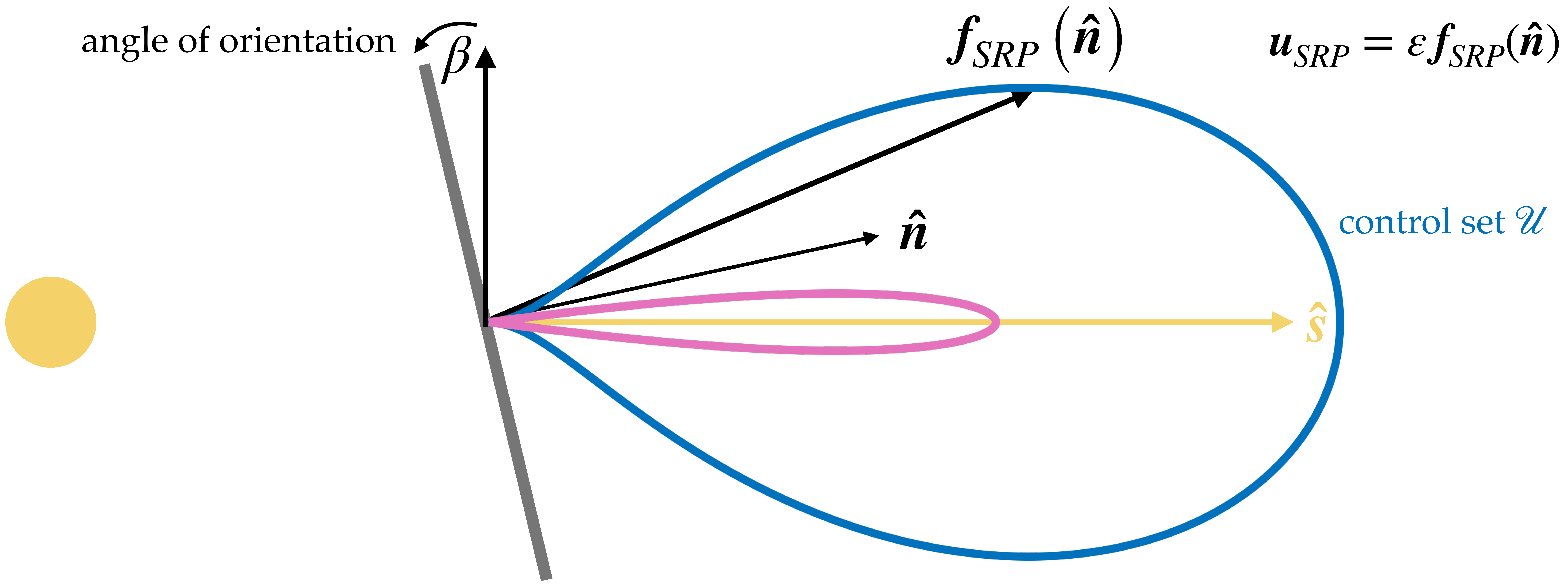
ESA contract no 4000134950/21/NL/GLC/my

Brijuni, August 2022

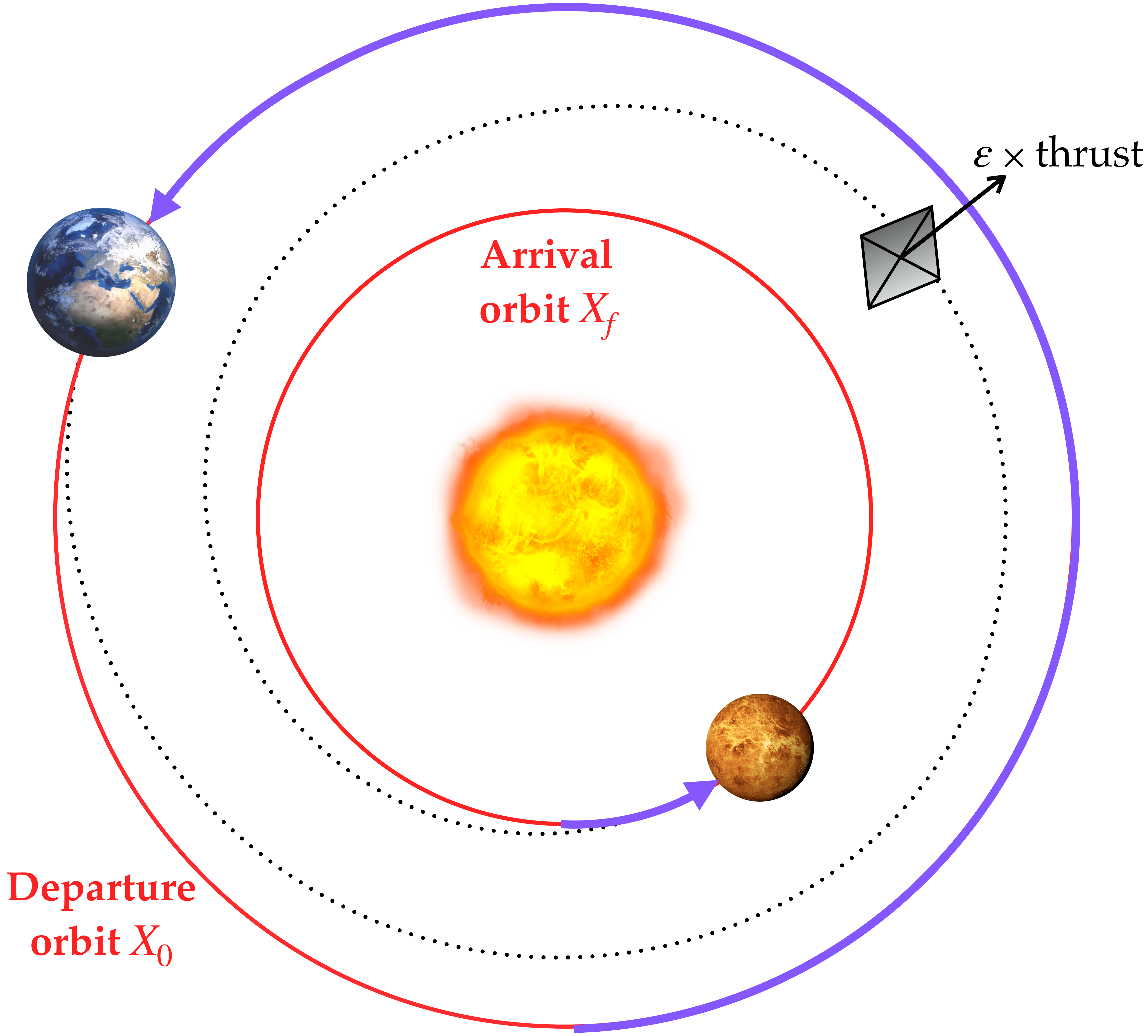
Solar sail



Control set


angle of orientation
β
$f_{SRP}\left(\hat{\boldsymbol{n}}\right)$
$\boldsymbol{u}_{SRP} = \varepsilon f_{SRP}(\hat{\boldsymbol{n}})$
$\hat{\boldsymbol{n}}$
control set $\mathscr{U}$
$\hat{s}$

Solar sail Lambert's problem



Given:

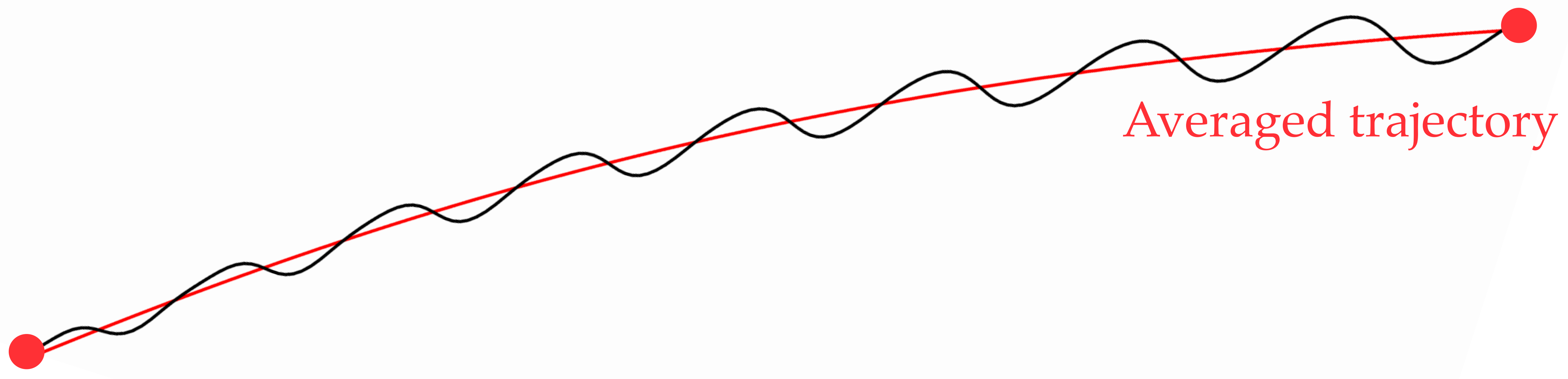
- Transfer time $t_f - t_0$
- Departure state, X_0 and $\varphi(t_0)$
- Arrival state, X_f and $\varphi(t_f)$

Find minimum ε-transfer

How averaging facilitates the solution

Smoothing: identification of local minima & robust convergence

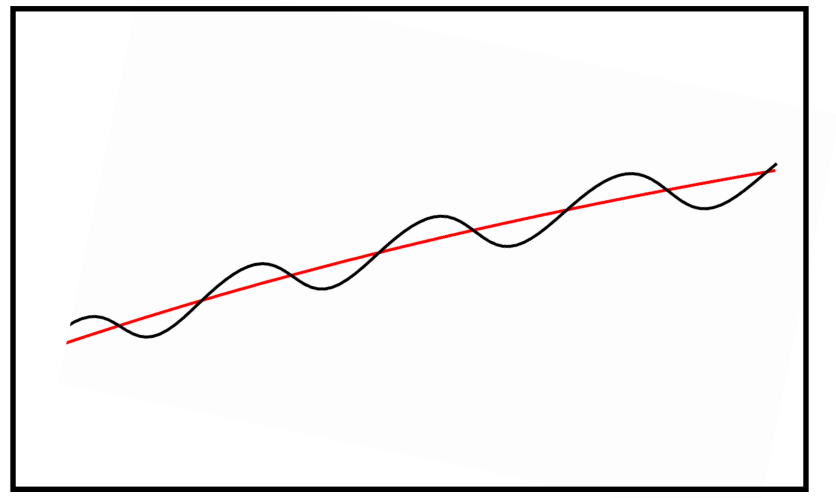
Reduced system: independent of fast variable φ → solution for any t_0 and t_f



Outline

min ε

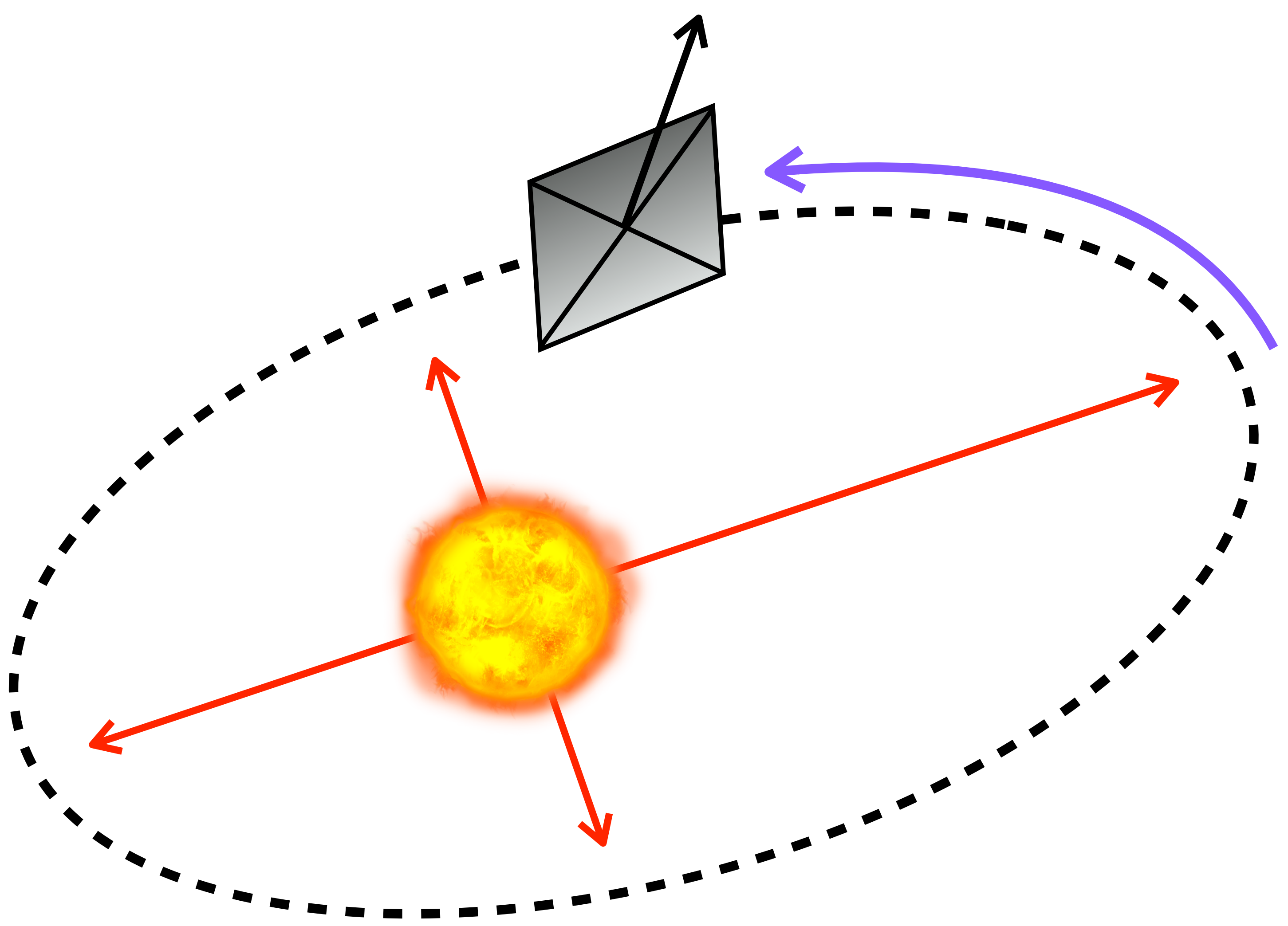
1. Necessary conditions for optimality



2. The averaged shooting problem and the algorithm

3. Results for Earth-Venus transfer

1. Optimal control problem



$$\min \varepsilon$$

subject to:

$$\frac{\mathrm{d}X}{\mathrm{d}t} = \varepsilon F(X, \varphi)u$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \omega(X) + \varepsilon G(X, \varphi)u$$

$$X(t_0) = X_0, \qquad \varphi(t_0) = \varphi_0$$

$$X(t_f) = X_f, \qquad \varphi(t_f) = \varphi_f$$

$$u \in \mathcal{U}, \qquad \forall t \in [t_0, t_f]$$

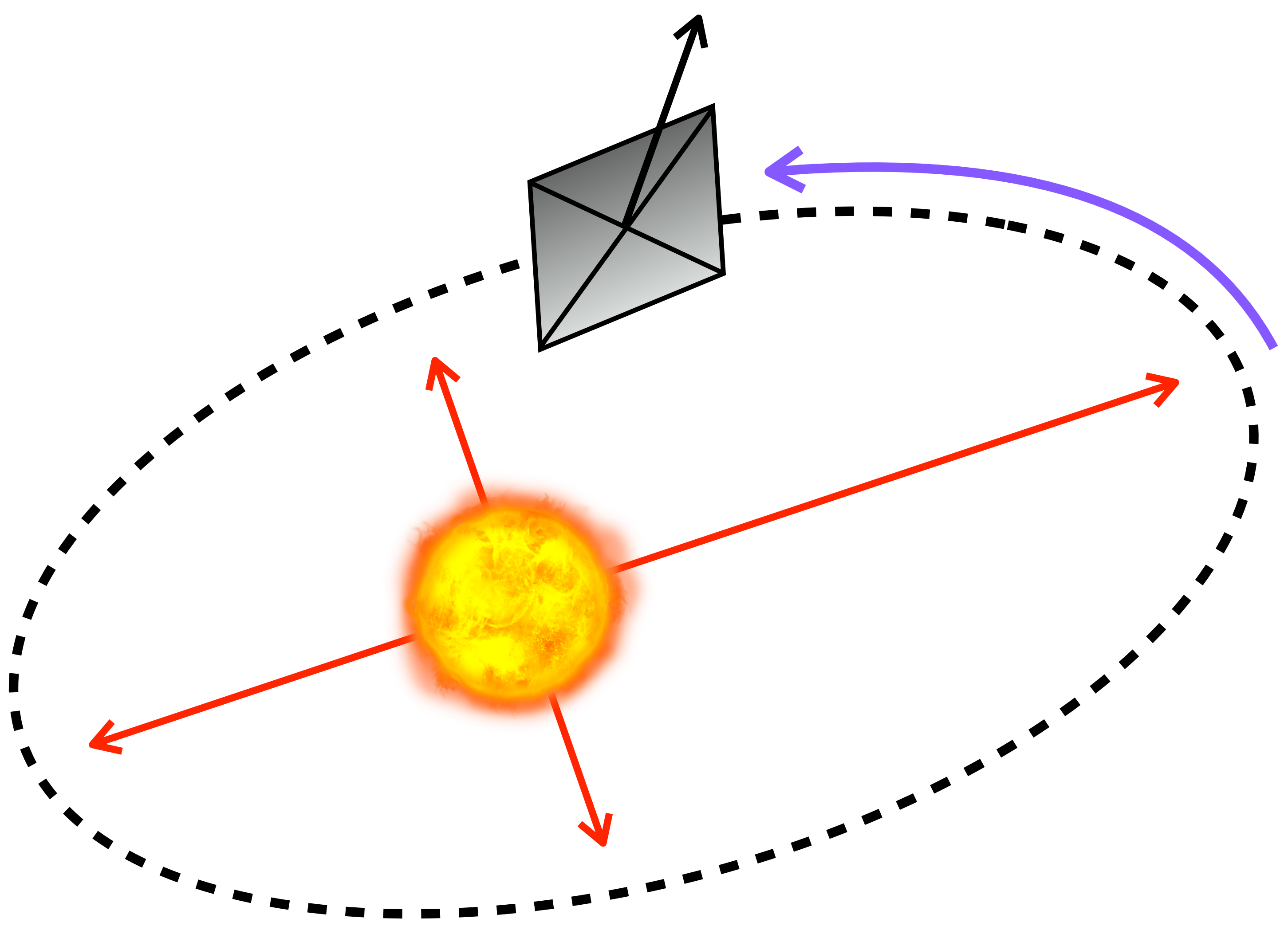
1. Necessary conditions for optimality

According to Pontryagin's maximum priciple (PMP), if u^* is optimal solution, then $\exists \, p_X$ the adjoint state s.t.

$$\dot{p}_X = -\nabla_X H, \quad \dot{X} = \nabla_{p_X} H,$$
$$u^* = \arg\max_{u \in U} H(X, p_X, u)$$

where $H(X, p_X, u)$ is the Hamiltonian of the system.

1. Dynamics of the system



Denote by p_X and p_φ the adjoints to X and φ

Hamiltonian:

$$\mathcal{H} = \omega(X)\, p_\varphi + \varepsilon \underbrace{\Big(\langle p_X F(X, \varphi), u^* \rangle + \langle p_\varphi G(X, \varphi), u \rangle\Big)}_{K(X, \varphi, p_X, p_\varphi)}$$

with $u^* := u^*(X, \varphi, p_X, p_\varphi)$

Boundary conditions:

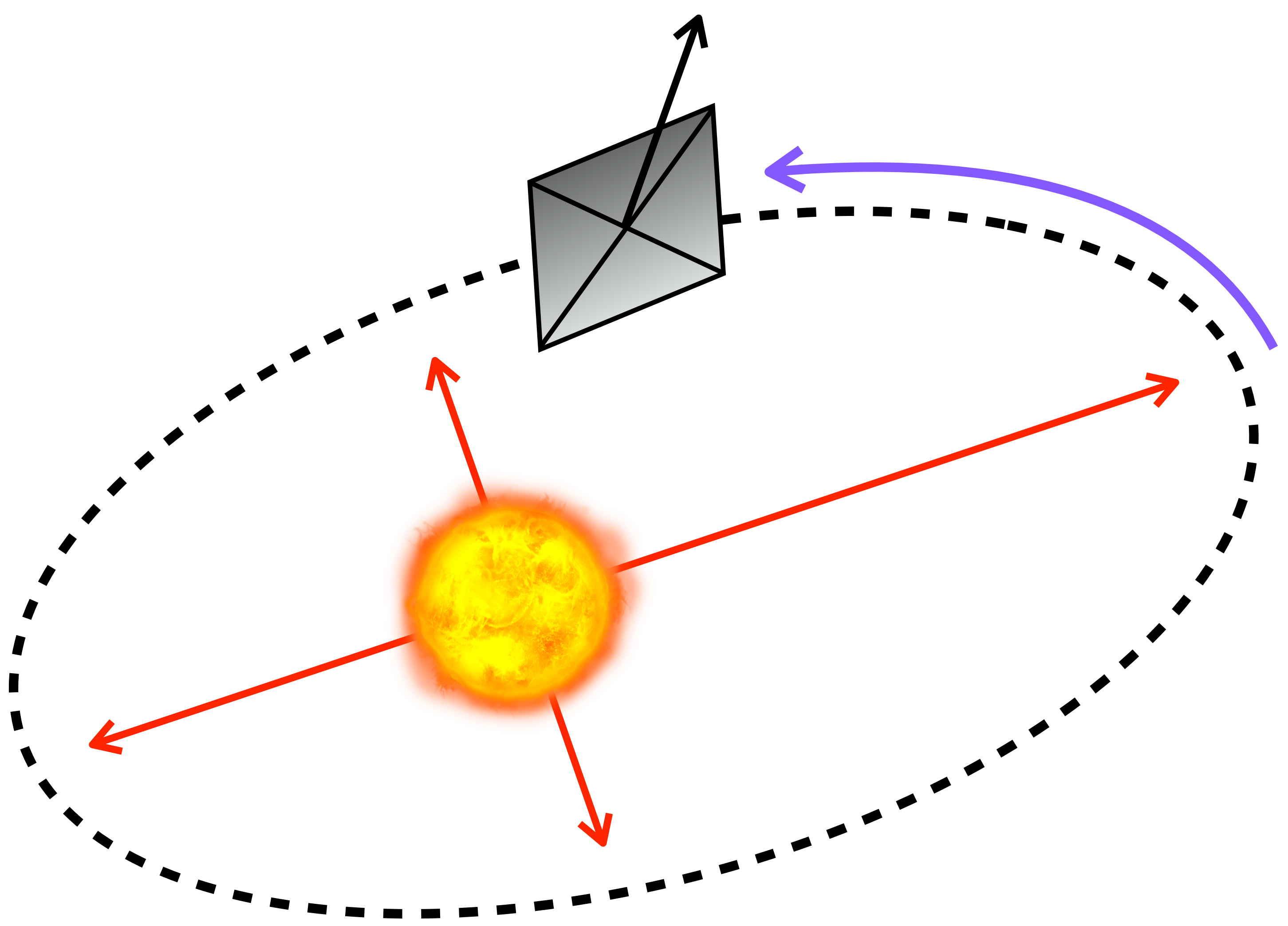
$$X(t_0) = X_0 \qquad \varphi(t_0) = \varphi_0$$

$$X(t_f) = X_f \qquad \varphi(t_f) = \varphi_f$$

Shooting variables:

$$\varepsilon, \quad p_X(t_0), \quad p_\varphi(t_0)$$

1. Averaging of the system



Averaged Hamiltonian:

$$\bar{\mathscr{H}} = \int_{\mathbb{S}^1} \omega(X)\, p_\varphi + \varepsilon \langle p_X F(X, \varphi), u \rangle + \varepsilon \langle p_\varphi G(X, \varphi), u \rangle \, \mathrm{d}\varphi$$

Rescaling:

$$\mathrm{d}\tau = \varepsilon \mathrm{d}t, \quad \psi = \varepsilon \bar{\varphi}, \quad p_\psi = \frac{\bar{p_\varphi}}{\varepsilon}$$

L. Dell'Elce, J.-B. Caillau, J.-B. Pomet. On the convergence of time-optimal manoeuvres of fast-oscillating control systems. European Control Conference, 2021.

2. The averaged shooting problem

	Original problem	**Averaged problem**
Shooting variables:	$\varepsilon, \quad p_X(t_0), \quad p_\varphi(t_0)$	$\tau_f, \quad \bar{p_X}(0)$
Hamiltonian:	$\mathscr{H} = \omega(X)\,p_\varphi + \varepsilon\, K(X, \varphi, p_X, p_\varphi)$	$\bar{\mathscr{H}} = \omega(\bar{X})\,p_\psi + K(\bar{X}, \bar{p_X})$
Boundary conditions:	X and φ are imposed at t_0 and t_f	$\bar{X}(0) = ?,\ \bar{X}(\tau_f) = ?,\ p_\psi(0) = p_\psi(\tau_f) = ?$

2. Algorithm to approximate solutions of the Lambert's problem

Step 1 Find a **one-parameter family of solution** of the average problem

$$\lambda \longrightarrow p_\psi, \bar{p}_X(0), \tau_f, \psi(\tau_f)$$

Step 2 Compute **sensitivities** of the average shooting function and short-periodic variations

$$\lambda, t_0, t_f \longrightarrow \delta\tau_f, \delta\psi_f$$

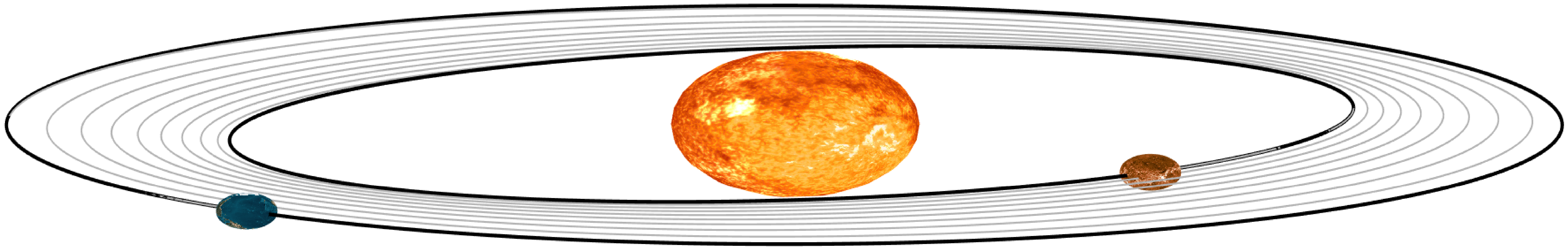
Step 3 Evaluate the **thrust magnitude** of each averaged solution

$$\varepsilon(\lambda, t_0, t_f) = \frac{\tau_f}{t_f - t_0 - \delta\tau_f}$$

Step 4 Identify solutions with the **correct boundary longitude**

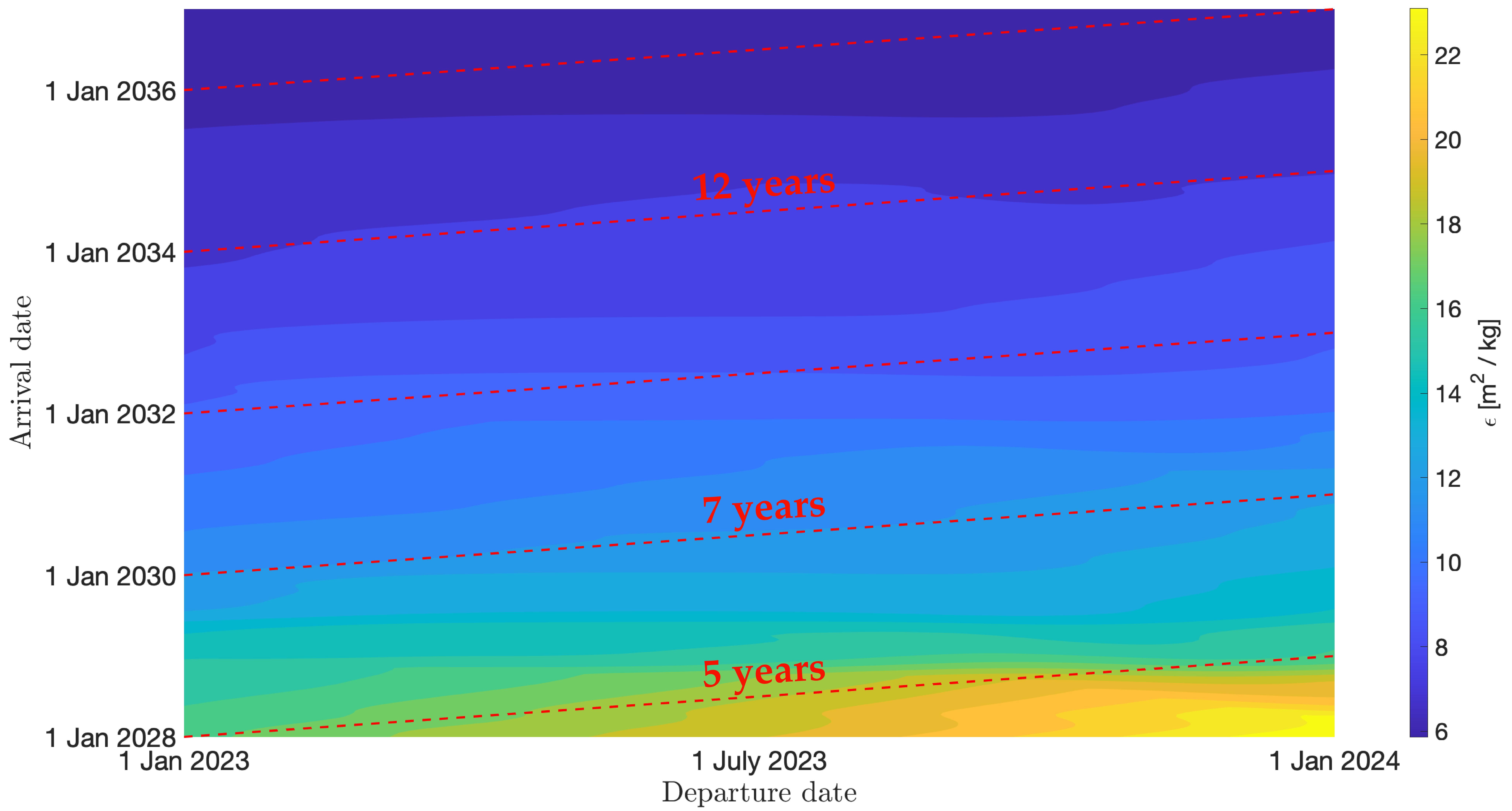
$$\forall t_0, t_f \quad \text{find } \lambda \quad \text{s.t.} \quad \text{mod}\left(\frac{\psi(\tau_f)}{\varepsilon} + \delta\psi_f + \varphi(t_0) - \varphi(t_f), 2\pi\right) = 0$$

3. Earth-Venus transfer: hight cost in inclination



	semimajor axis (AU)	orbital period (years)	eccentricity	inclination to elliptic (deg)
Earth	1	1	0.017	0
Venus	0.7233	0.6152	0.007	3.39

3. Earth-Venus transfer: a 'porkchop chart'



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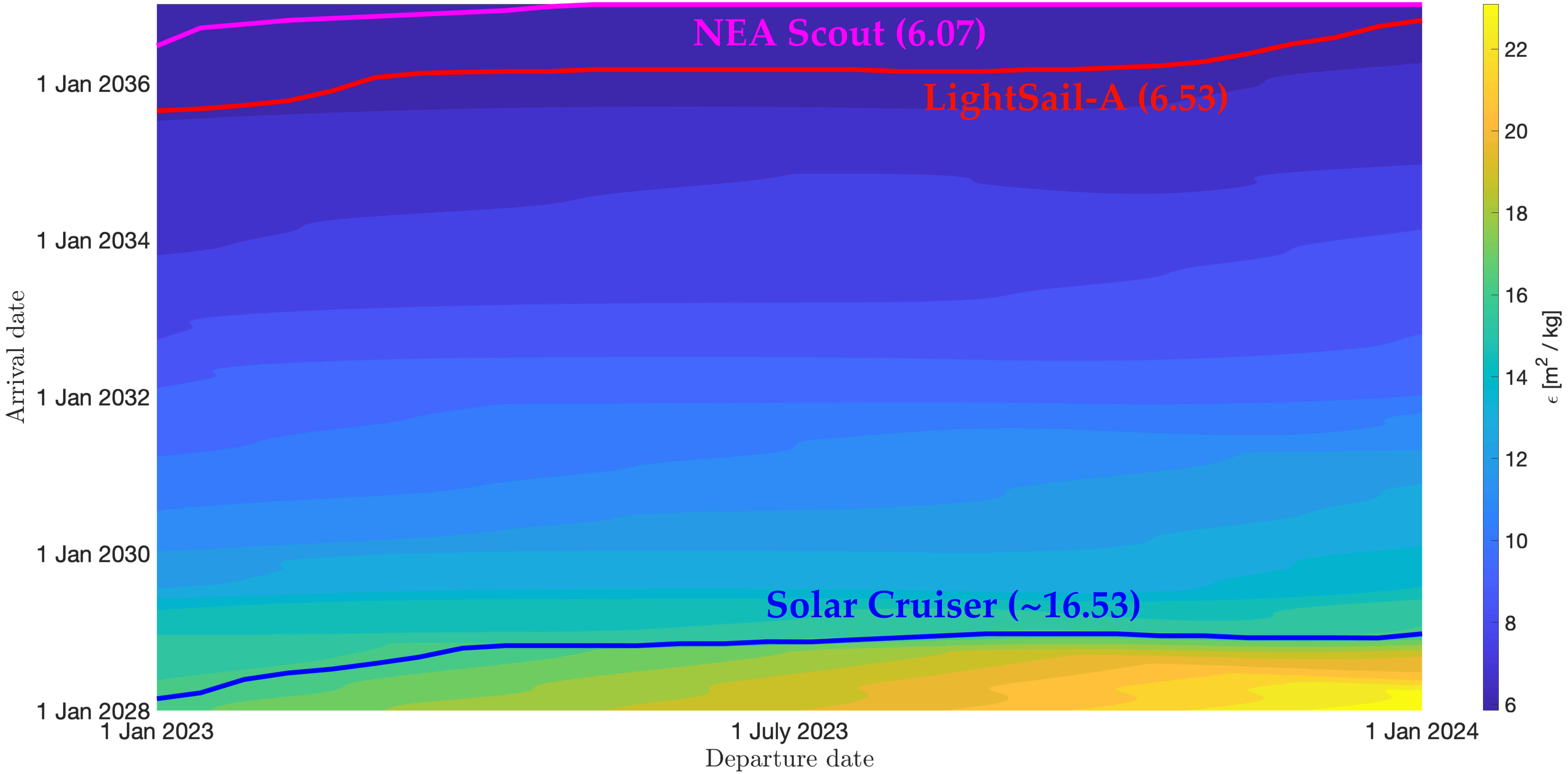

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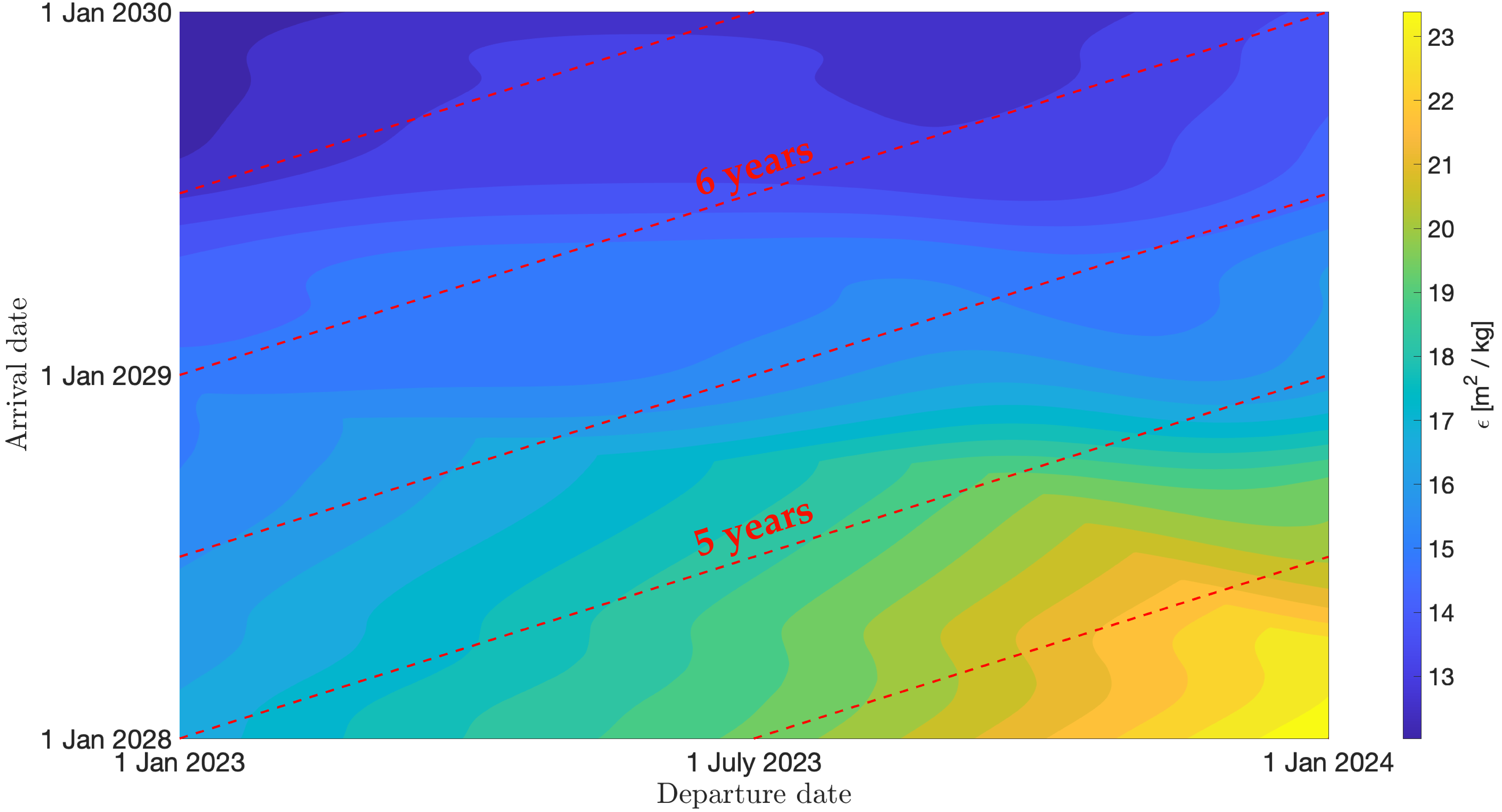
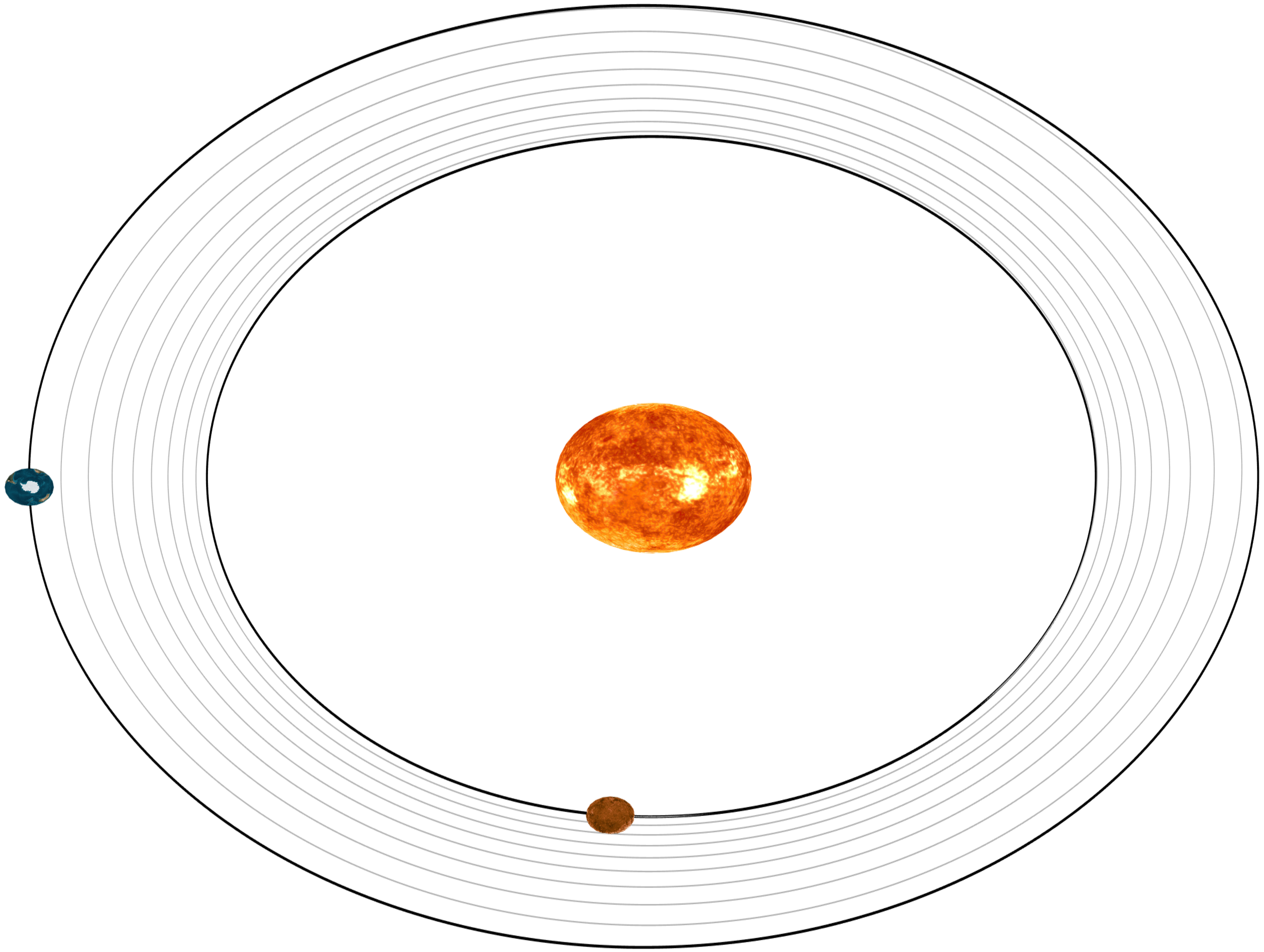


3. Earth-Venus transfer: 7 years, $\varepsilon = 12.17$



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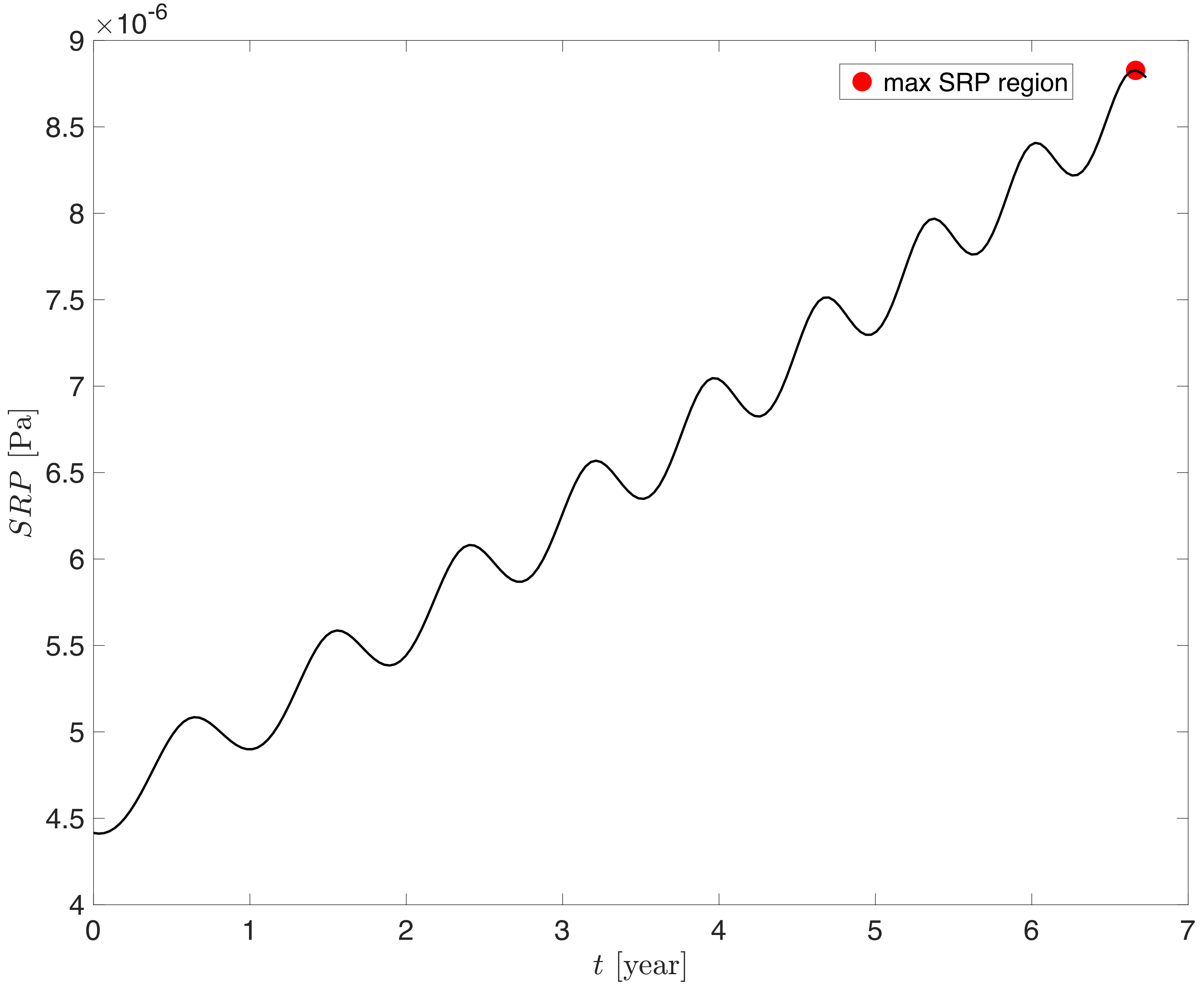
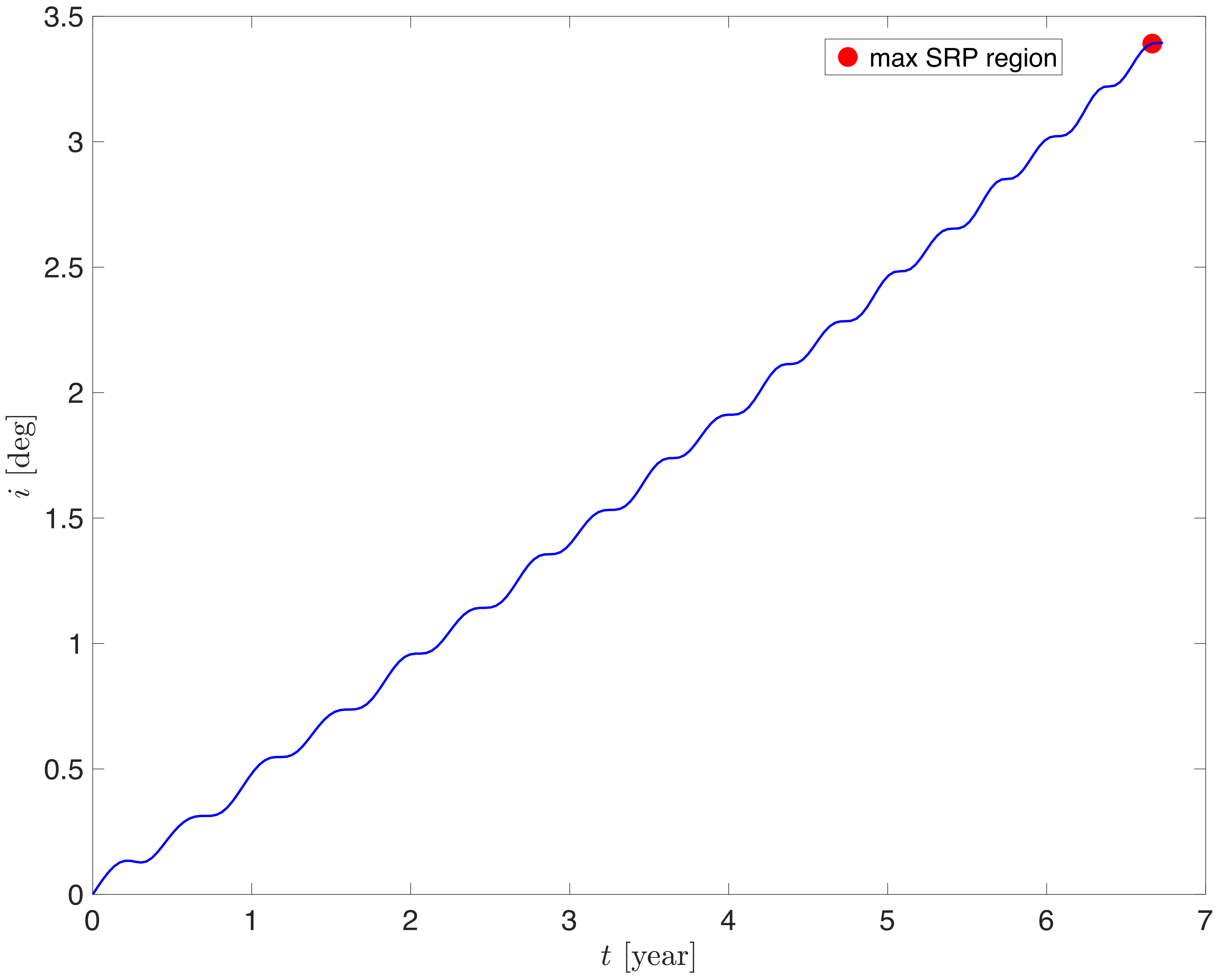
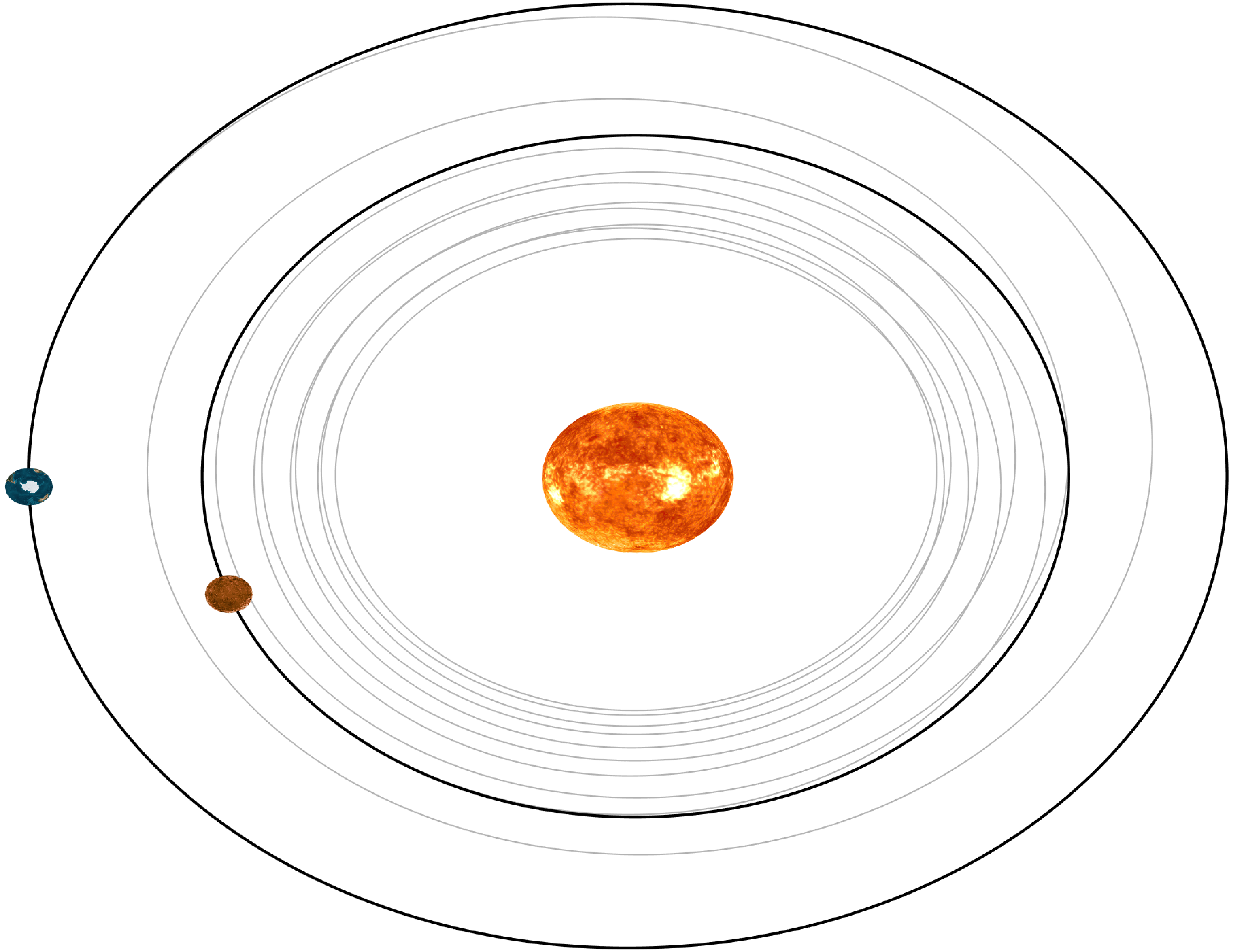




3. Earth-Venus transfer: 5 years, $\varepsilon = 18.90$



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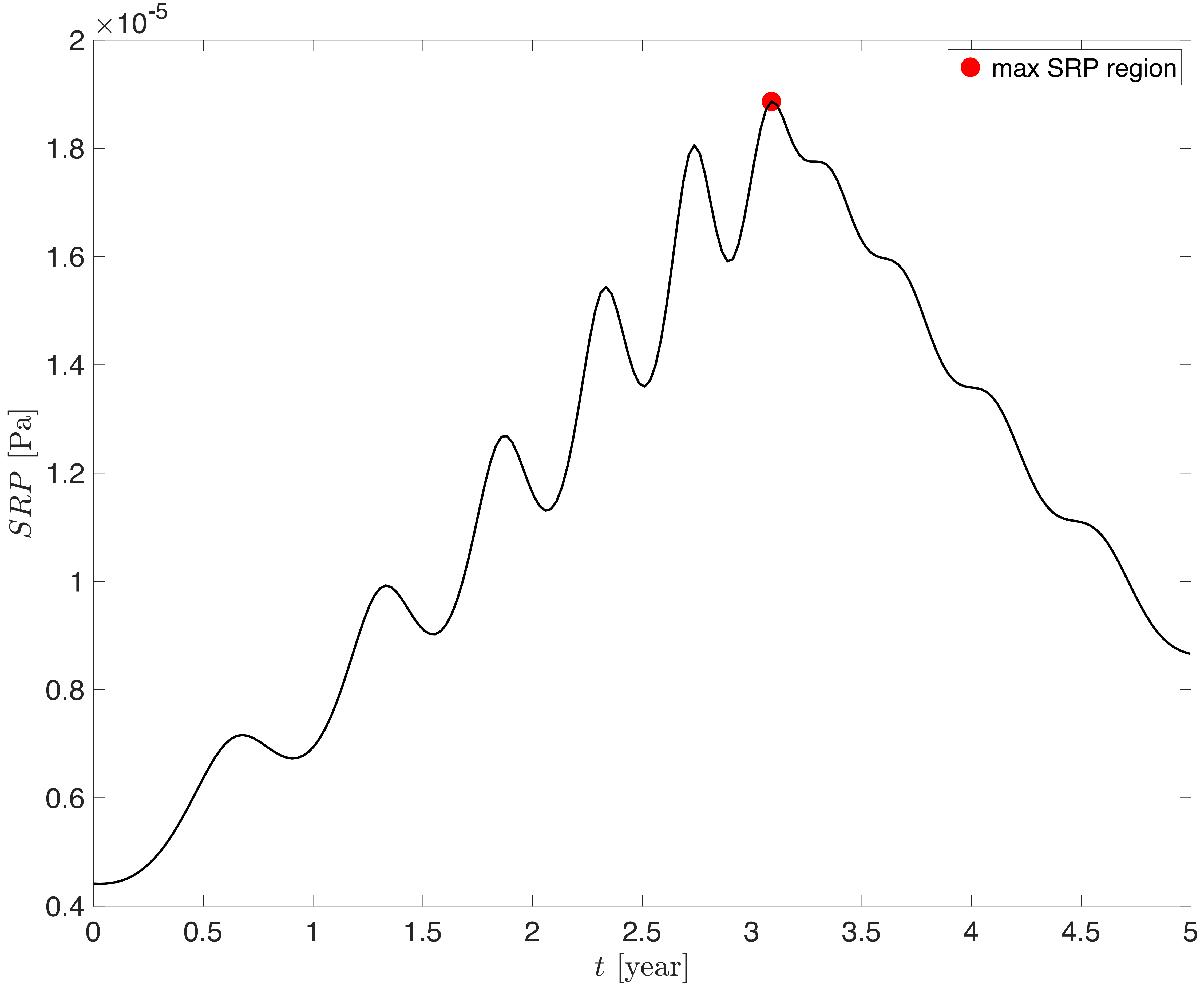
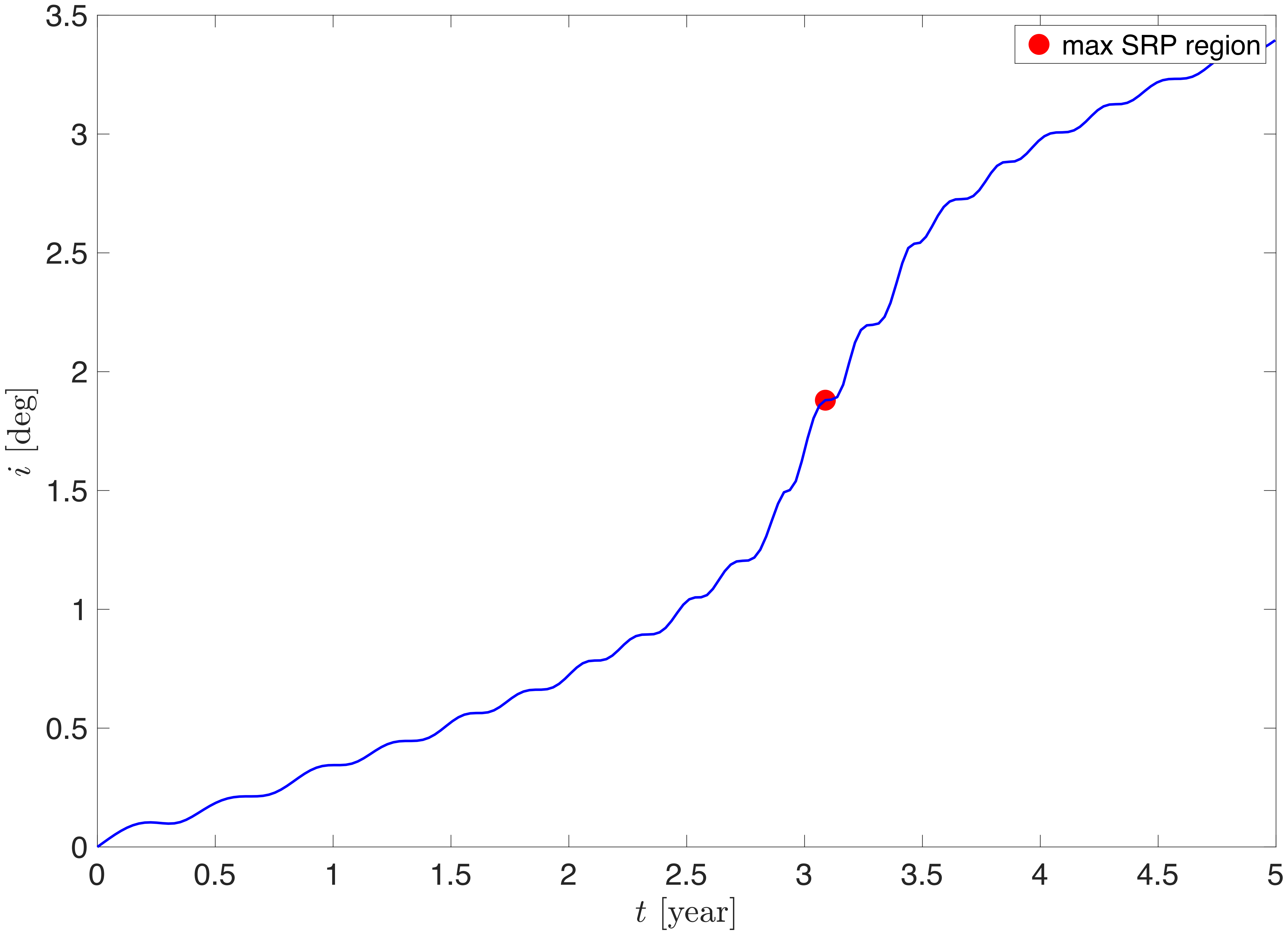
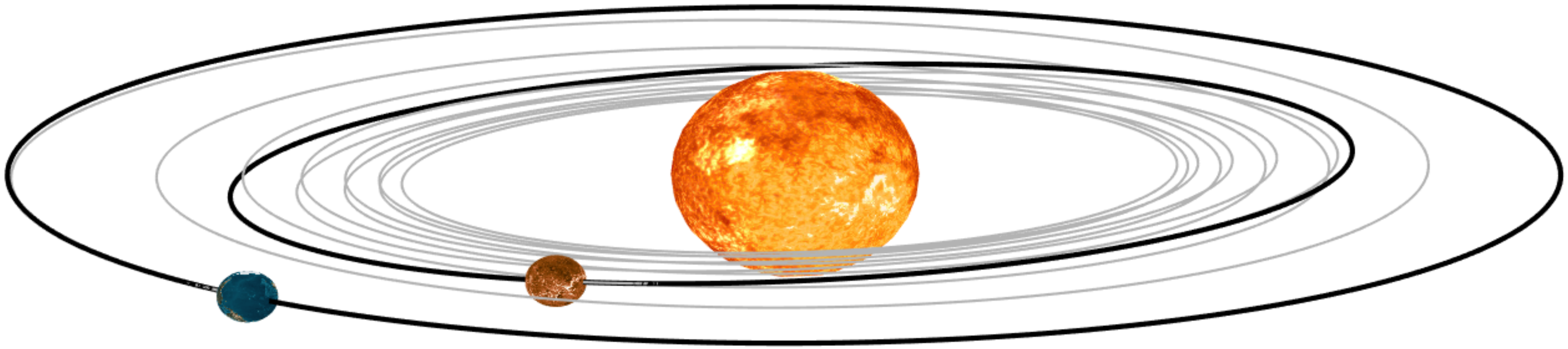




Conclusion

The algorithm offers reliable initial guess for the original problem for any departure/arrival windows.

Solar sails offer different ways for interplanetary transfert depending on its physical properties.



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