Cold Atoms: from Hubbard models to quantum computing

- A colloquium style introduction …
  - engineering of Hubbard models & applications
  - quantum computing

- Present research topics
  - single atom transistors
  - quantum optical / solid state interfaces

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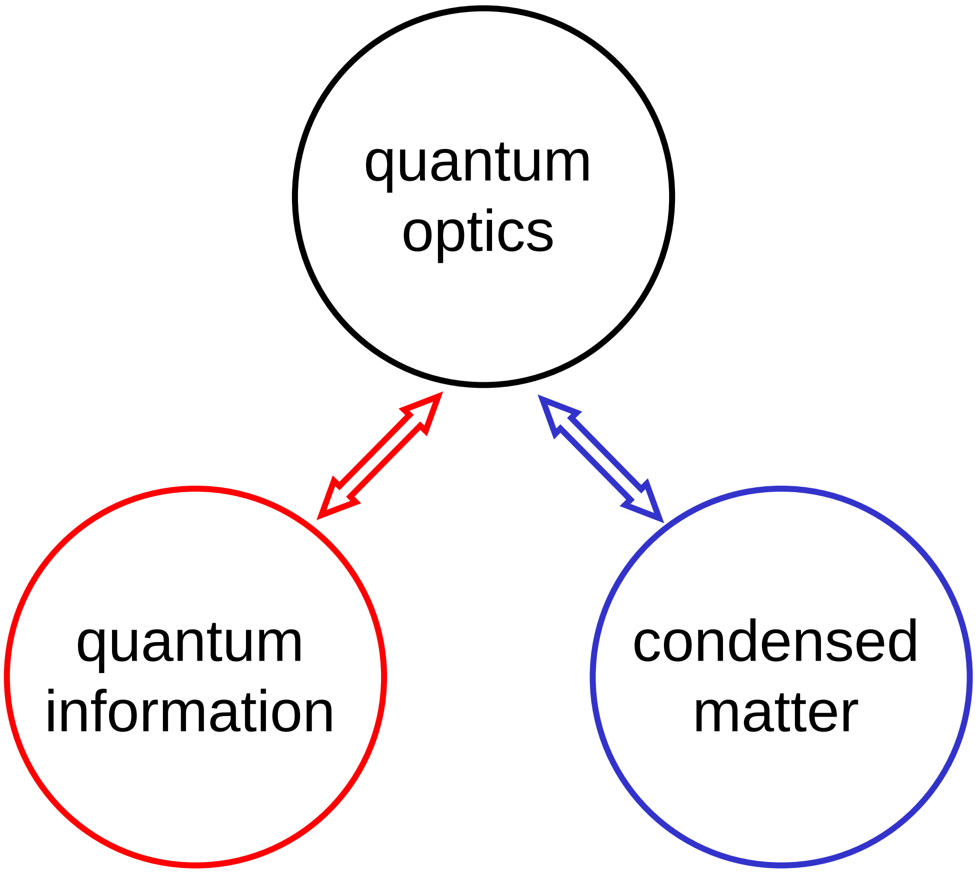
SFB
Coherent Control of Quantum Systems

€U TMR networks

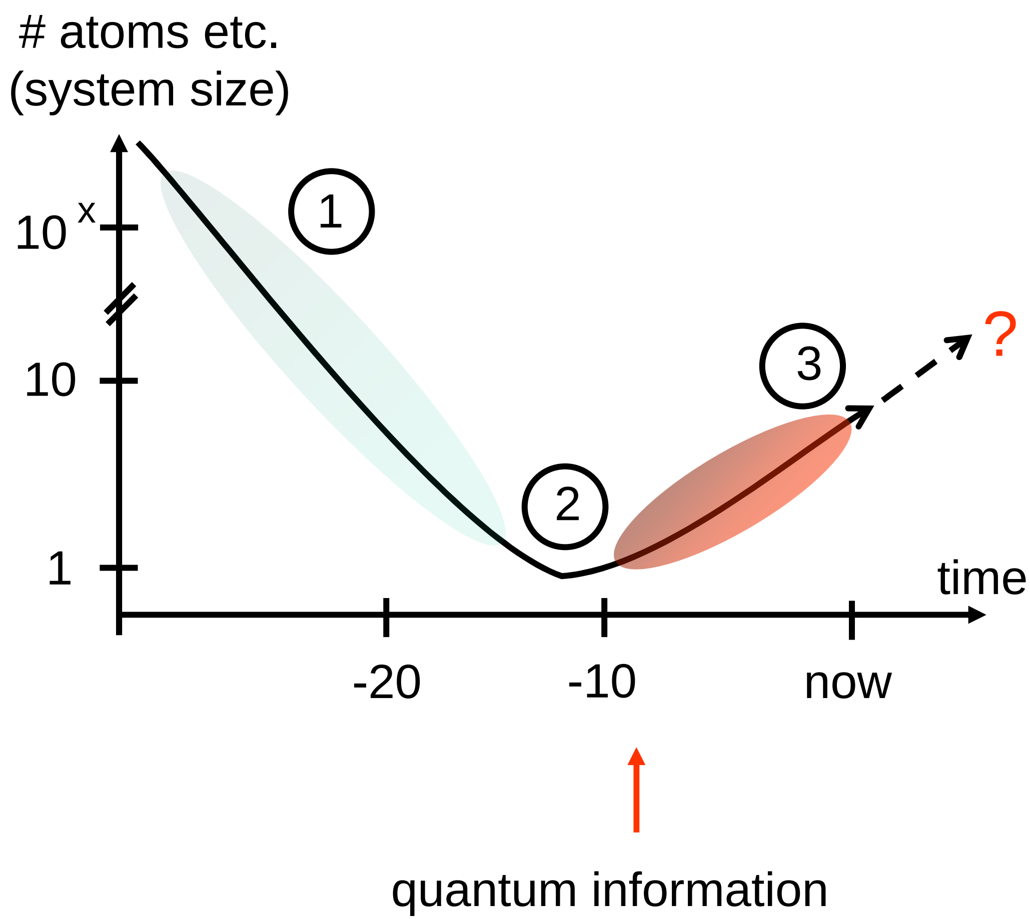
Institute for Quantum Information

Outline



Quantum Optics

- systems: atoms, ions, photons, CQED, …
- … driven by experimental progress
- … provided theoretical framework



1. early days:
- N>>1 (large number)
- dissipation dominated

  example: laser physics

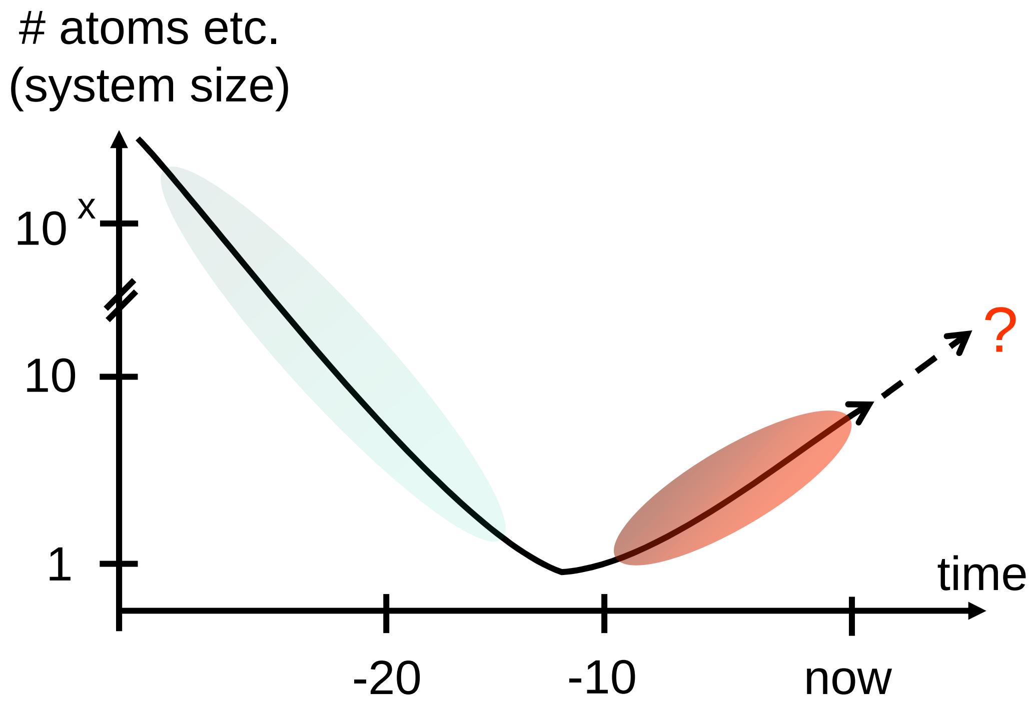
2. single atoms, ions & photons
- preparation (cooling, trapping)
- coherent interactions
  >> dissipation
- measurement

3. composite systems:
- many atoms etc.
- …

motivation
- fundamental quantum physics
- applications
  - ✓ precision measurement
  - ✓ quantum information

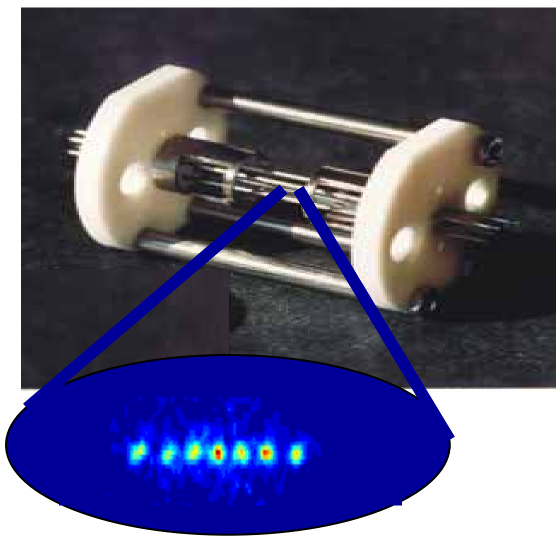
Quantum Optics

- systems: atoms, ions, photons, CQED, …
- … driven by experimental progress
- … provided theoretical framework

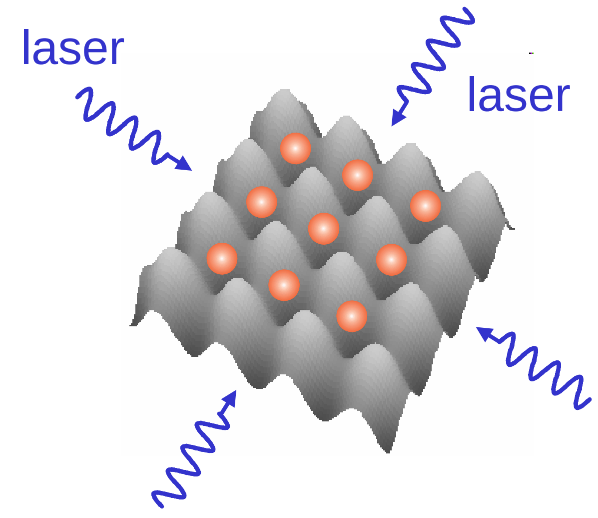


Examples:

- trapped ions

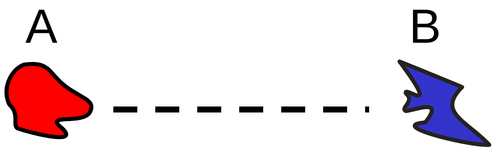


- optical lattices



Entangled States

- entanglement



states: $|0\rangle \otimes |0\rangle$

$|1\rangle \otimes |1\rangle$ ... product states

but also ...

$$\frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)$$ ... entangled

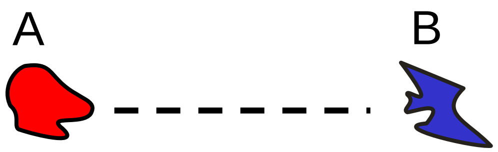
Schrödinger:
Verschränkung

- fundamental aspects of quantum mechanics
- applications
  - quantum communication & computing
  - precision measurement

Engineering Entangled States

We need ...

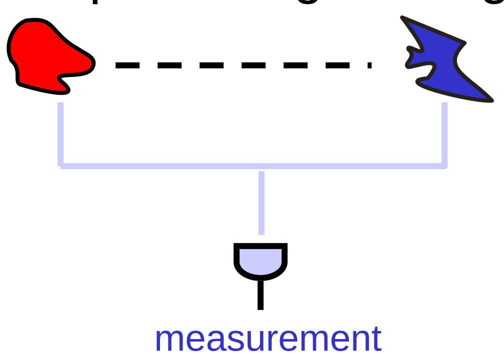
- “quantum engineering”



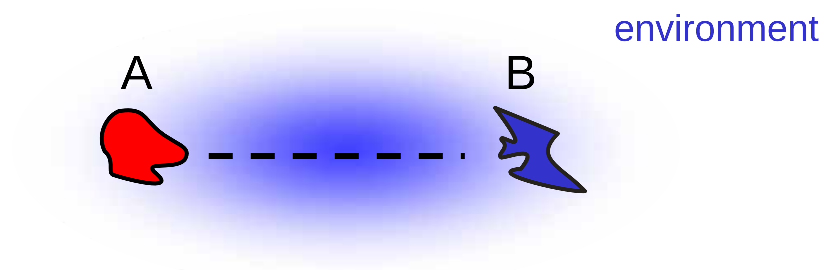
$$|a\rangle_A |b\rangle_B \rightarrow \sum c_{ab} |a\rangle_A |b\rangle_B$$

Hamiltonian evolution

- or: “quantum gambling”



- isolation

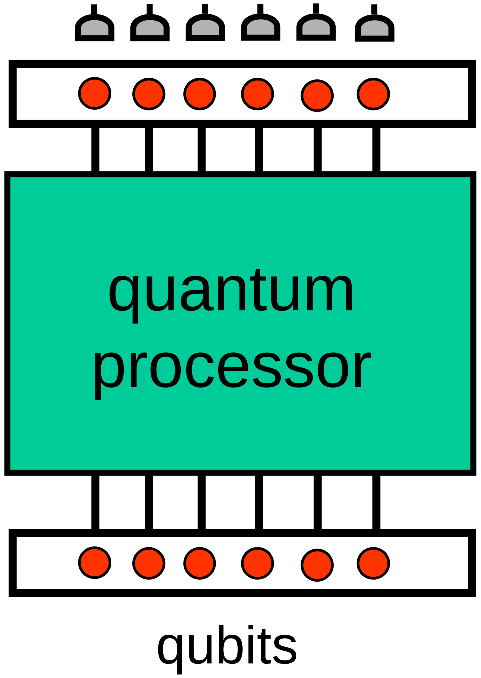


$$|\phi\rangle_A |\varphi\rangle_B |E\rangle \rightarrow |\Psi\rangle_{ABE}$$

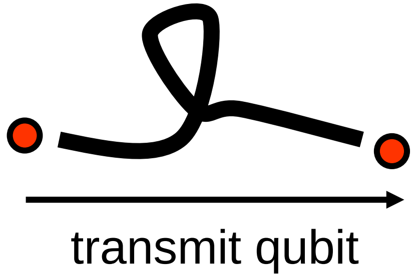
$$\rho_{AB} = \mathrm{tr}_E |\Psi\rangle_{ABE} \langle \Psi| \neq |\Psi\rangle_{AB} \langle \Psi|$$

Quantum optical systems provide one of the best set-ups to create entangled states in a controlled way.

quantum computing



quantum communication

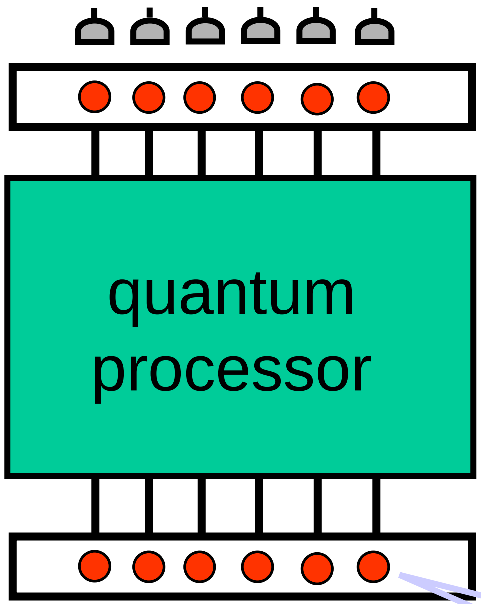


information science:

- qualitatively new concepts of computation
- new efficient algorithms
- new complexity classes (?)

- quantum cryptography

quantum computing



qubits

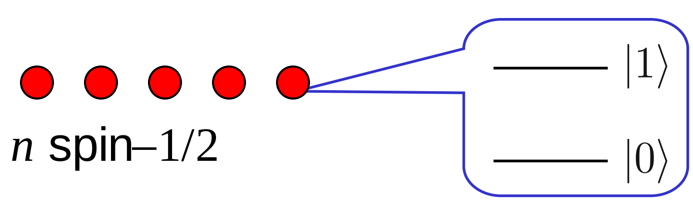
$$|\psi_{\text{out}}\rangle$$

$$|\psi_{\text{out}}\rangle = \hat{U}|\psi_{\text{in}}\rangle$$

$$|\psi_{\text{in}}\rangle = \sum_x c_x |x_{N-1}, \ldots, x_0\rangle$$

quantum register

- quantum memory: qubits

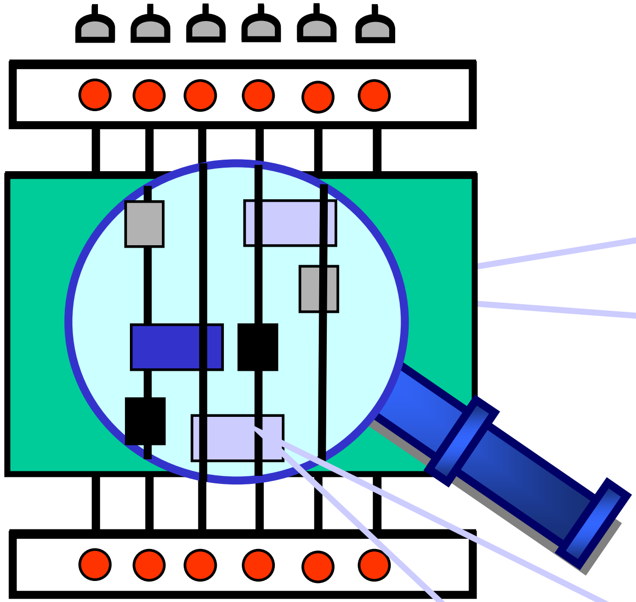


n spin–1/2

example: two qubit entangled state

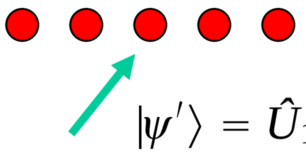
$$|\psi\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

quantum computing



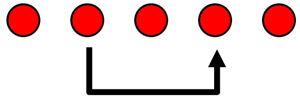
quantum gates

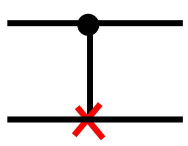
- single qubit gate



$$|\psi'\rangle = \hat{U}_1|\psi\rangle$$

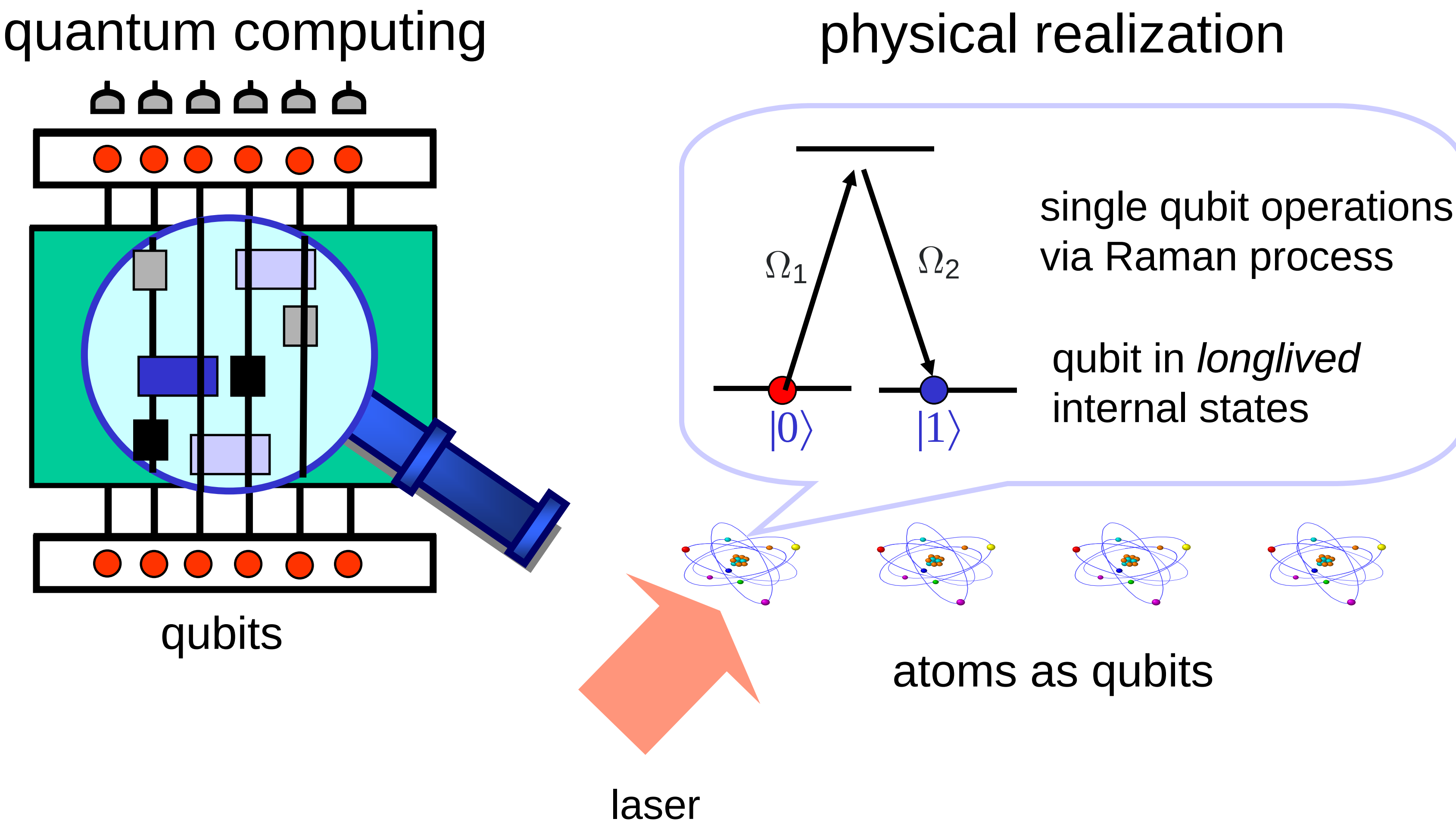
- two qubit gate: entanglement





truth table

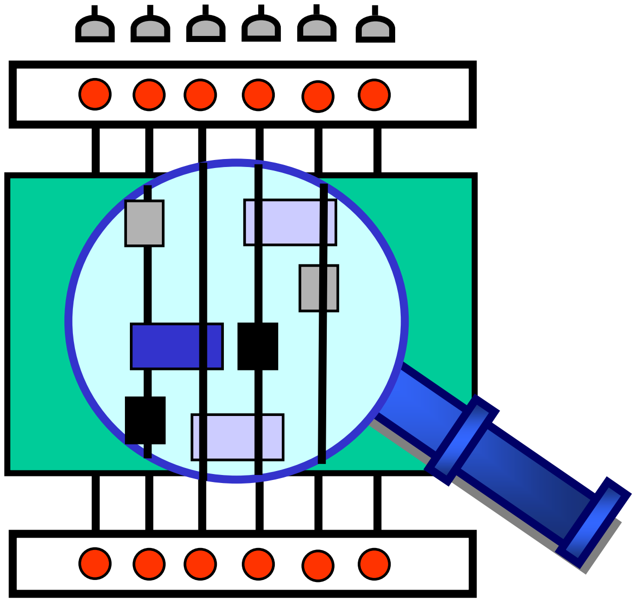
$|0\rangle|0\rangle$	$\rightarrow$	$|0\rangle|0\rangle$
$|0\rangle|1\rangle$	$\rightarrow$	$|0\rangle|1\rangle$
$|1\rangle|0\rangle$	$\rightarrow$	$|1\rangle|1\rangle$
$|1\rangle|1\rangle$	$\rightarrow$	$|1\rangle|0\rangle$



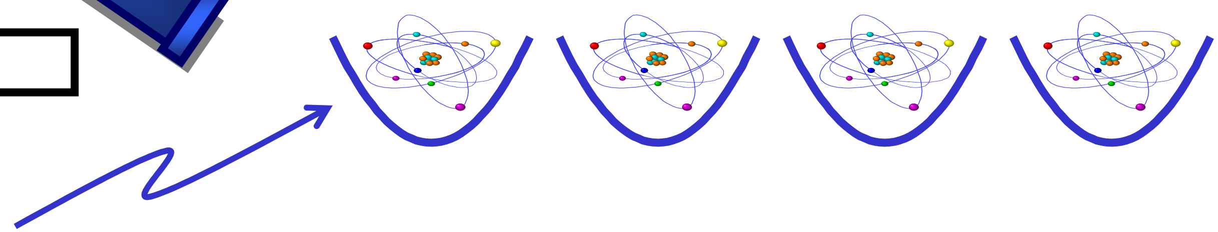
Requirements:

addressing
single qubit

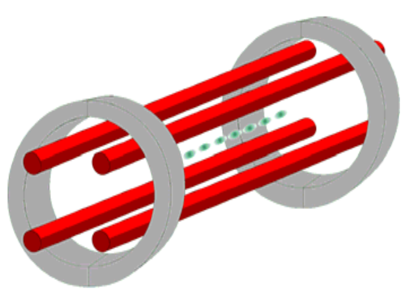
trapping the qubit



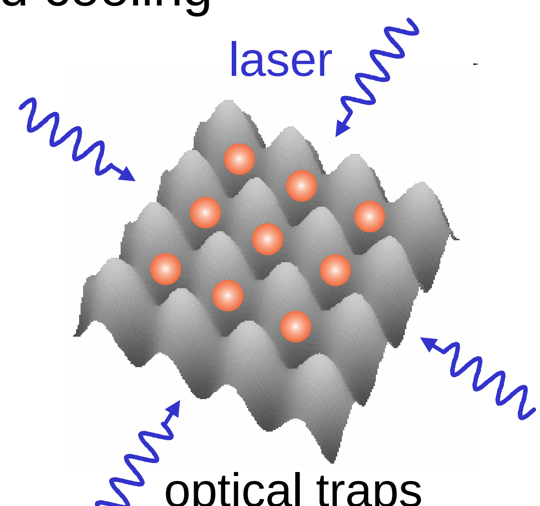
qubits



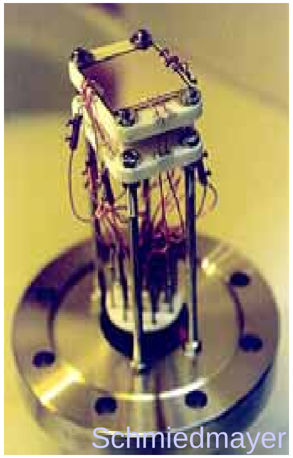
- atomic trapping and cooling



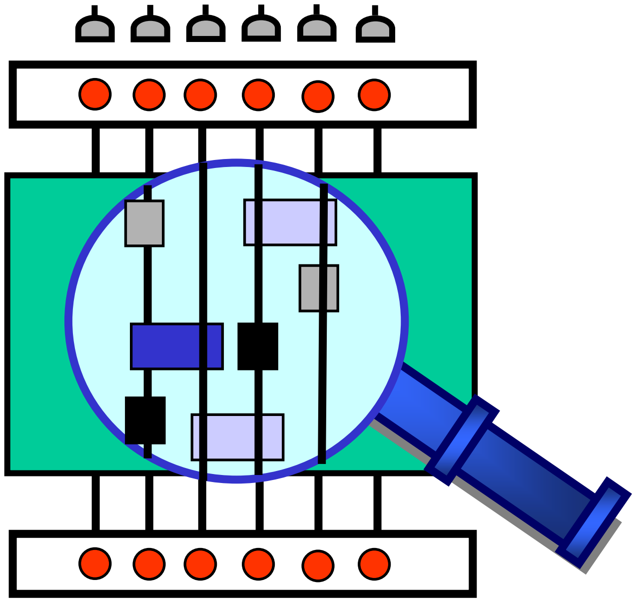
ion trap



optical traps

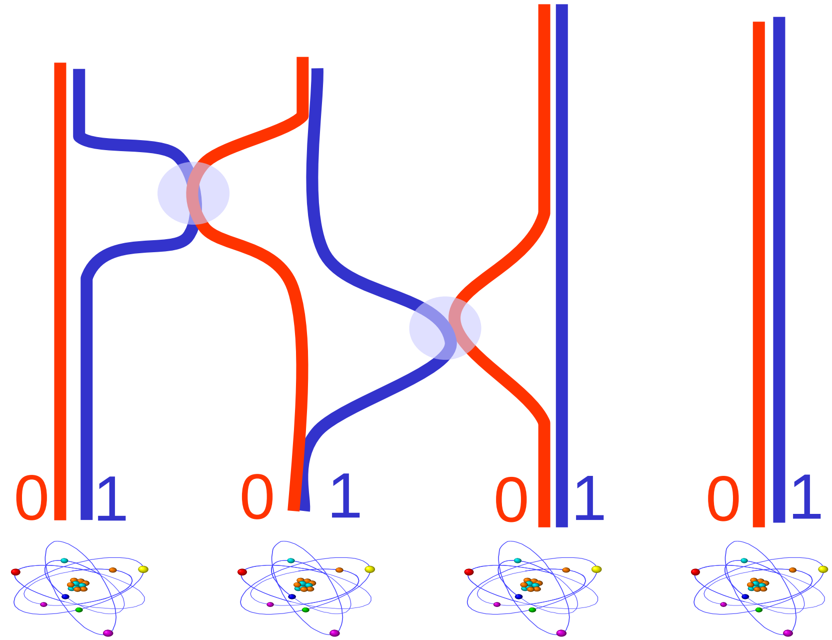


magnetic traps

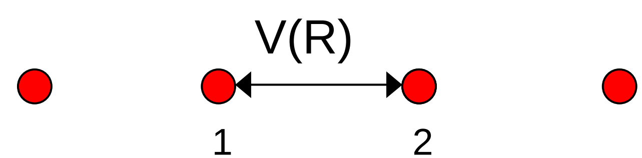


qubits

entangling qubits



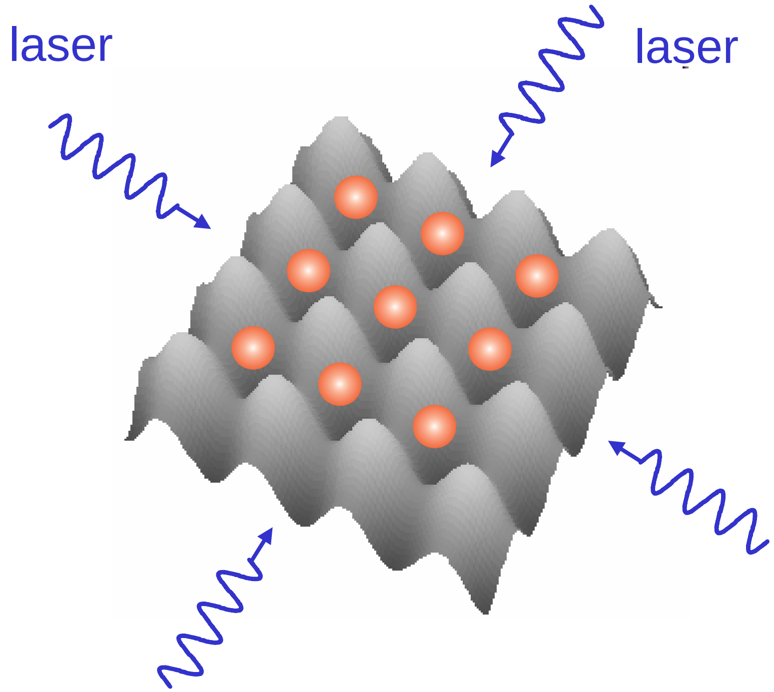
e.g. controllable two body interactions:
controlled collisions, ...



Hamiltonian $H = \Delta E(t)|1\rangle_1\langle 1|\otimes|1\rangle_2\langle 1|$

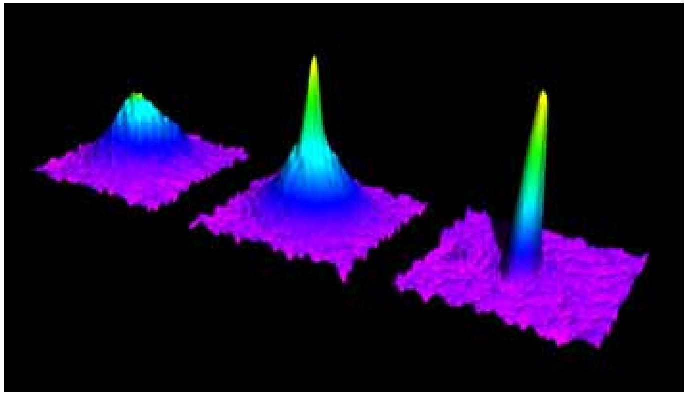
so that $|1\rangle_1|1\rangle_2 \longrightarrow e^{i\phi}|1\rangle_1|1\rangle_2$

Neutral Atoms (in optical lattices)



- *large* arrays of atoms / qubits
- manipulation of atoms / qubits: controlled interactions
- how?
  - ✓ preparation
  - ✓ controlled interaction
    - kinetic engery vs. interaction
    - entanglement

- **BEC as a weakly interacting gas**



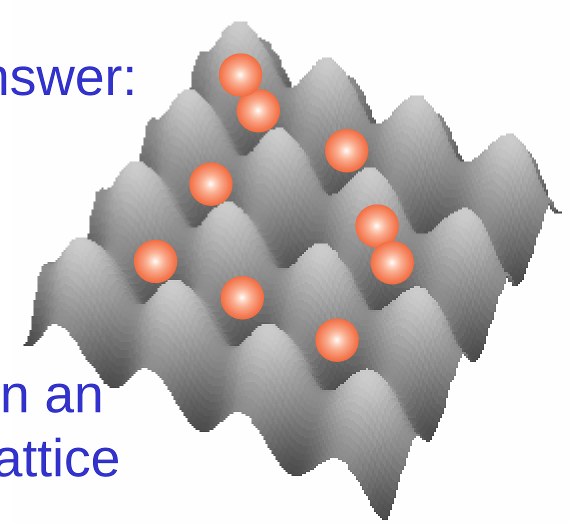
- **condensed matter physics**

✓ frontier: strongly interacting systems

✓ Question: *Strongly interacting* quantum system with *dilute* quantum gases?

kinetic energy >> interactions → ? kinetic energy ~ interactions

✓ Answer:

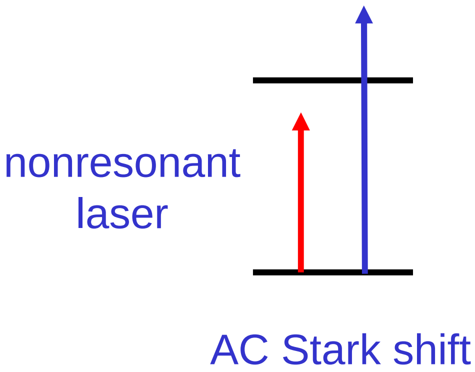


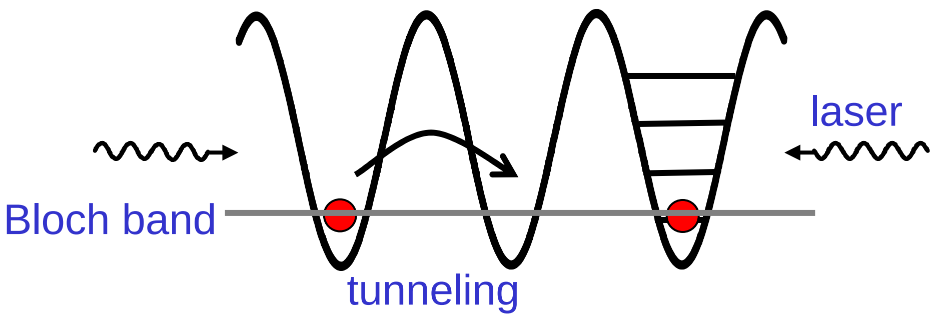
atoms in an optical lattice

Gross-Pitaevskii Equation → ~~Gross-Pitaevskii Equation~~

Optical lattice

- optical lattice as array of microtraps

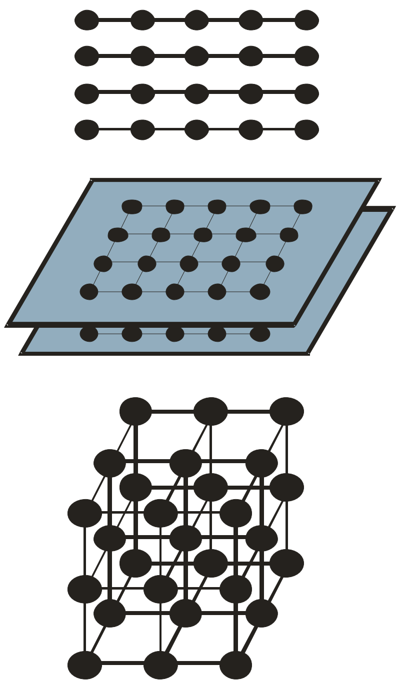




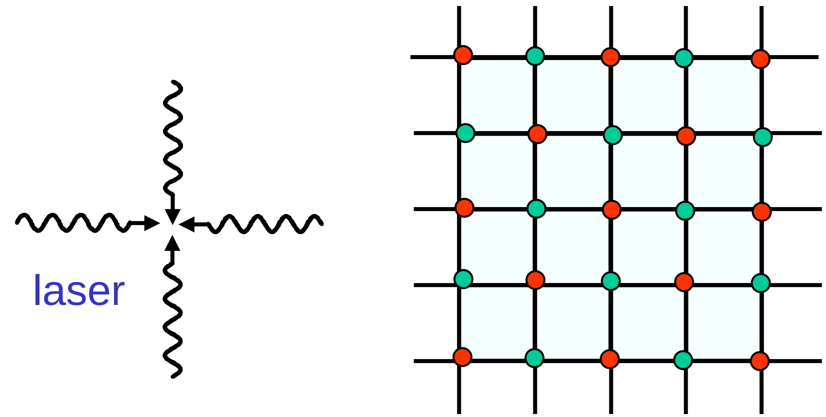
optical lattice as array of microtraps

Building lattice models

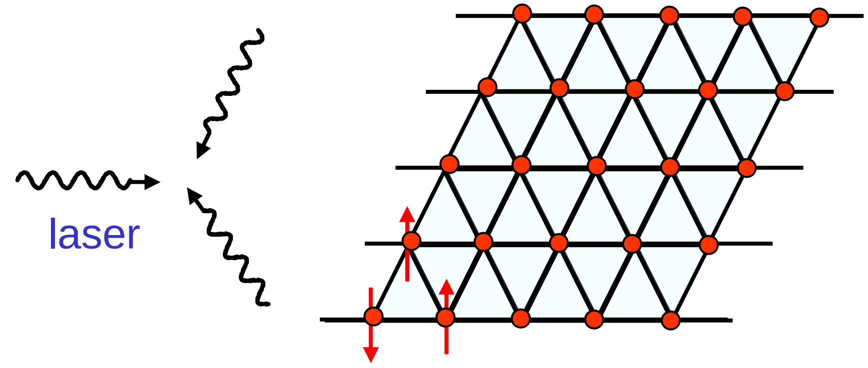
- 1D, 2D and 3D



- lattice configurations



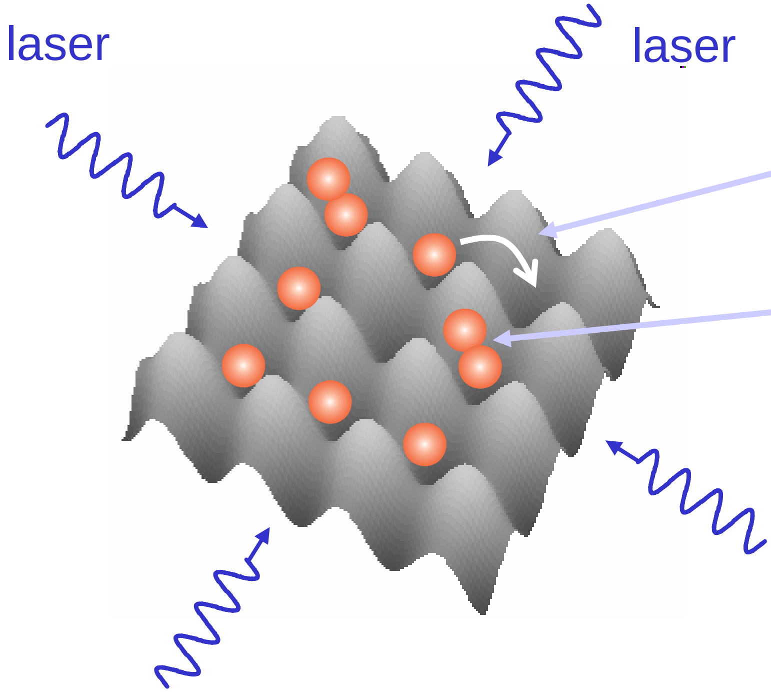
square lattice



triangular lattice

- random potentials …
- effective magnetic fields …
- time dependent

Cold atoms in an optical lattice

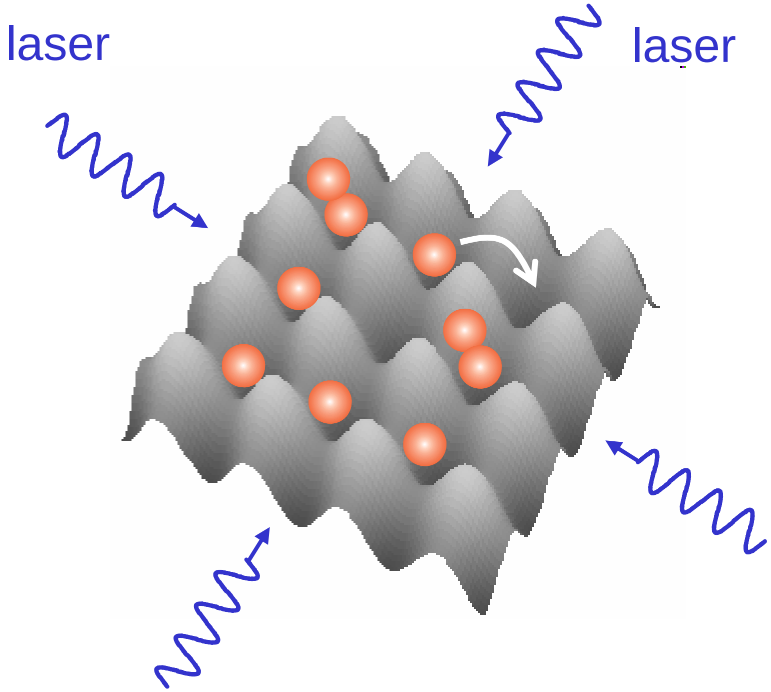


- dynamics

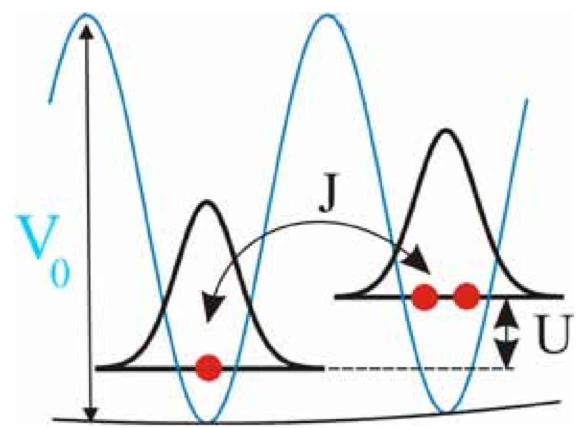
✓ hopping between lattice sites: kinetic energy

✓ onsite interaction due to repulsion: potential energy

Cold atoms in an optical lattice



- Bose Hubbard model



$$H = -\sum_{\alpha \neq \beta} J_{\alpha\beta} b^\dagger_\alpha b_\beta + \frac{1}{2} U \sum_\alpha b^\dagger_\alpha b^\dagger_\alpha b_\alpha b_\alpha$$

kinetic engery: hopping

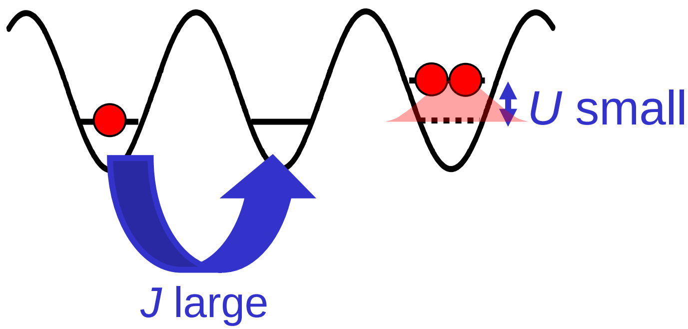
interaction: onsite repulsion

b_α ... bosonic destruction operator at site α

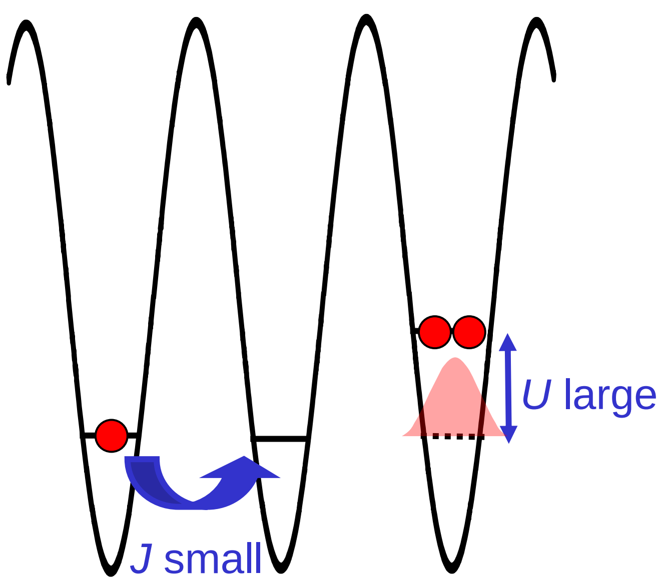
Innsbruck 1998

Laser control: kinetic vs. potential energy

- **shallow lattice** : weak laser



- **deep lattice:** intense laser



weakly interacting system:

J >> U

(kinetic energy >> interactions)

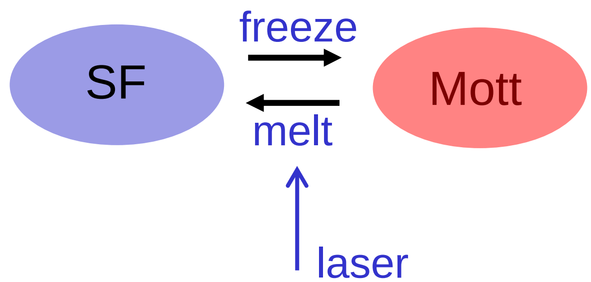
laser parameters
(time dependent)

strongly interacting system:

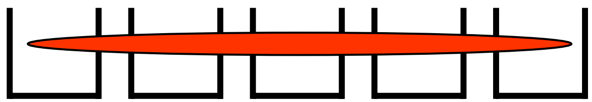
J << U

(kinetic energy << interactions)

Superfluid – Mott insulator *quantum* phase transition



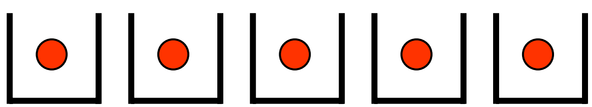
- superfluid J>>U



delocalized atoms: BEC

$$\left(b_1^\dagger + \ldots + b_M^\dagger\right)^N |\text{vac}\rangle$$

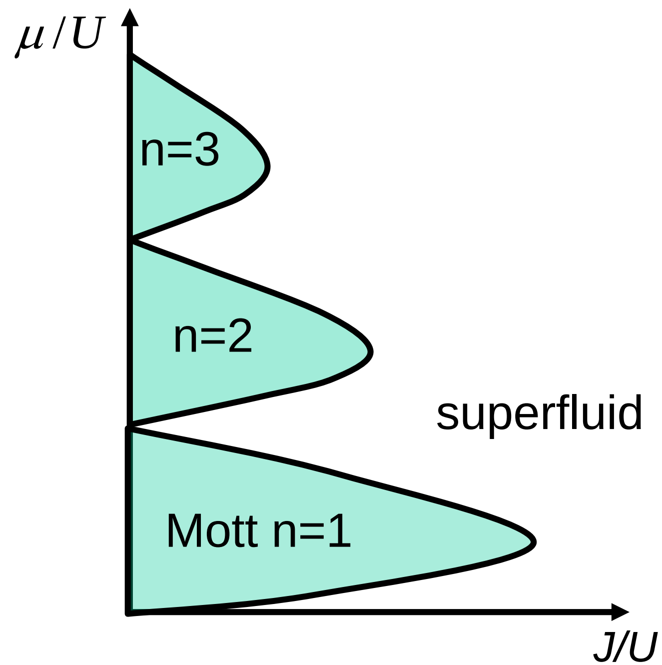
- Mott phase: J<<U



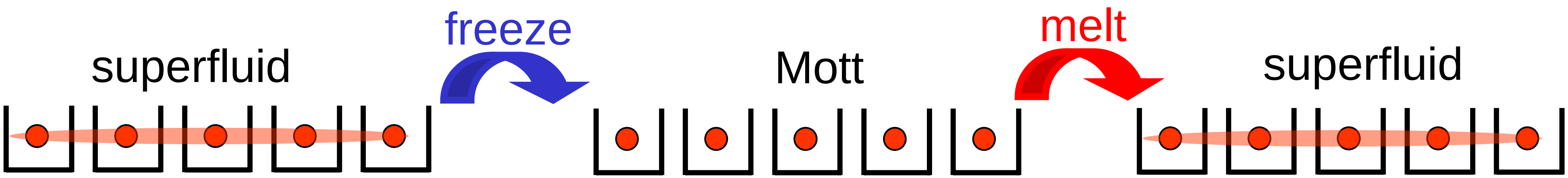
$$b_1^\dagger b_2^\dagger \ldots b_M^\dagger |\text{vac}\rangle$$

"Fock states"

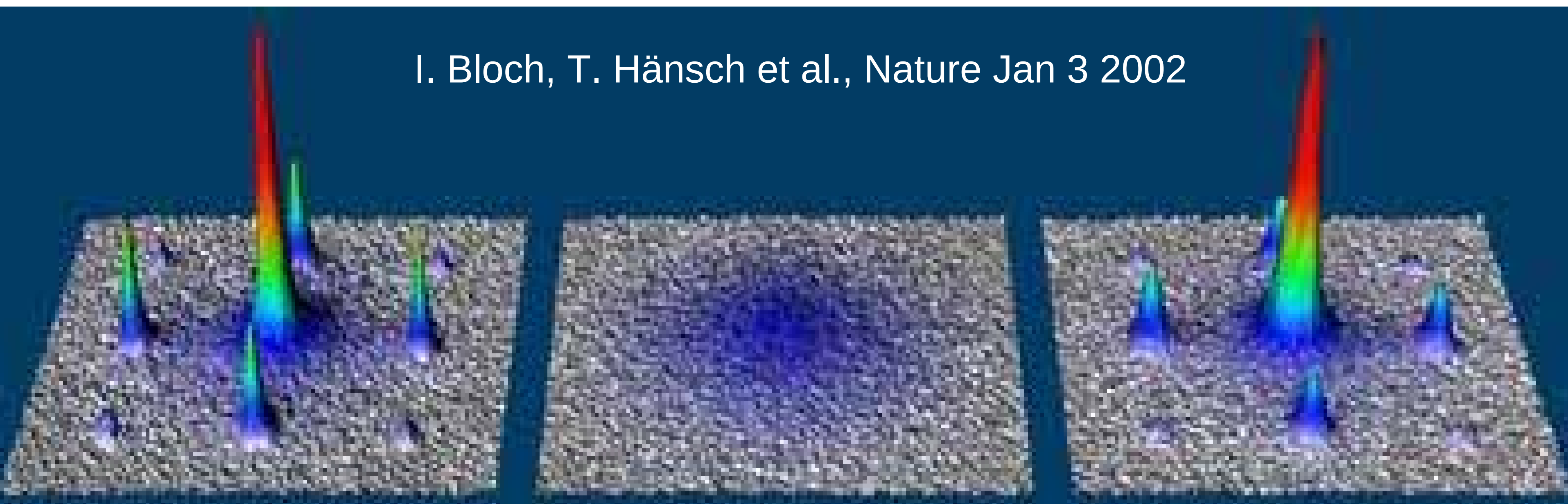
- phase diagram



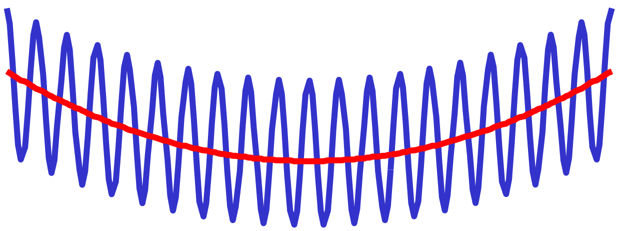
regular filling with exactly 1, 2 or 3 atoms per lattice site

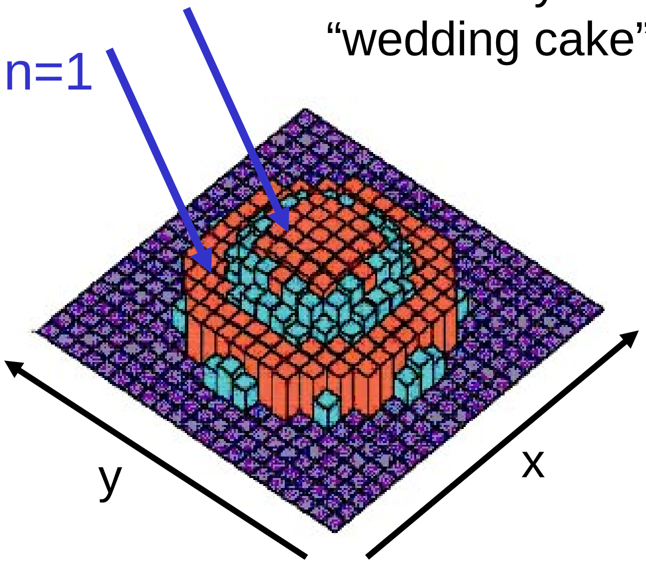


interference NO interference interference

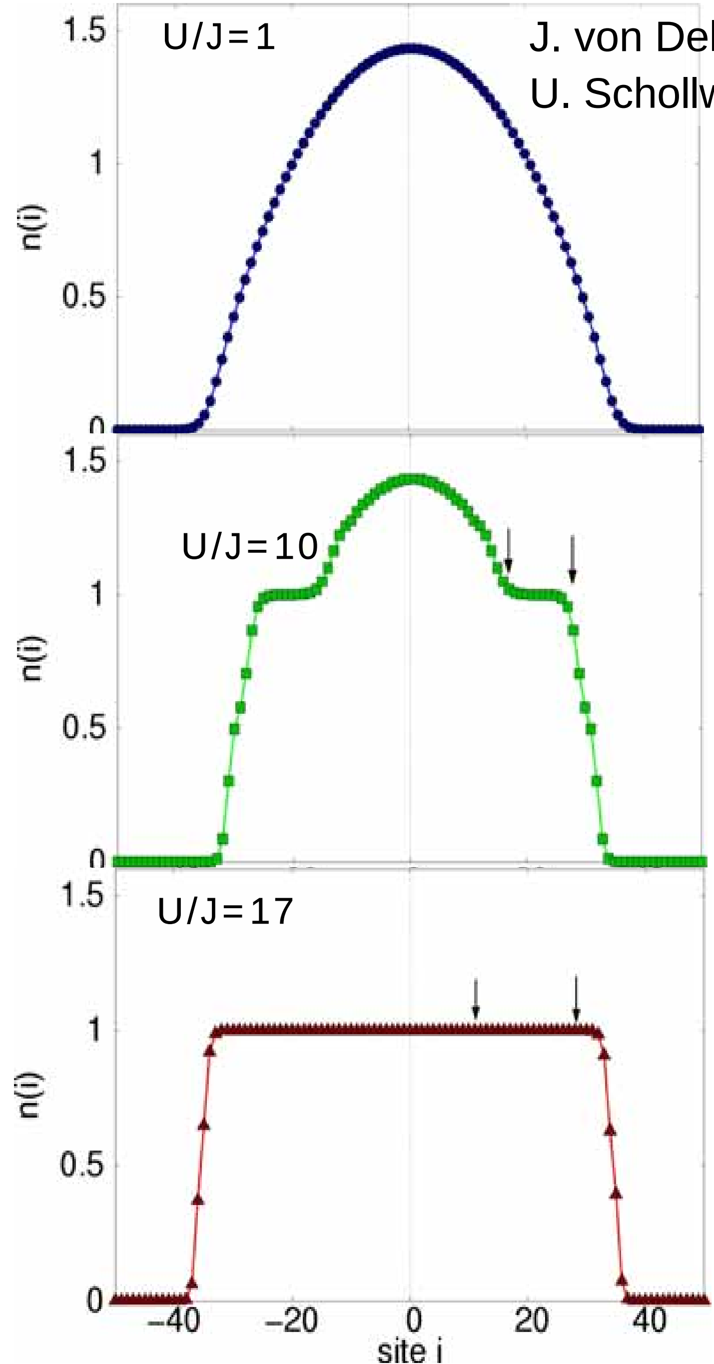


Superfluid and Mott in 2D trap

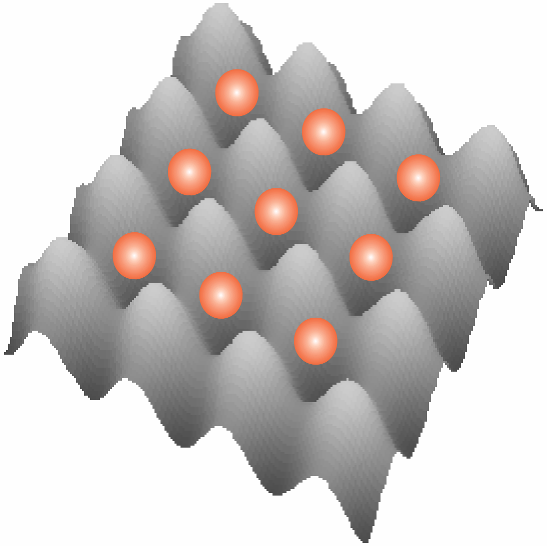




C. Kollath
J. von Delft
U. Schollwöck



A first summary



What we have achieved so far …

1. quantum lattices gases
   - strongly interacting
   - tunable parameters

2. load LARGE arrays of qubits in optical lattices via Mott insulator transition

Building condensed matter simulators … (where present CM theory provides no answer)

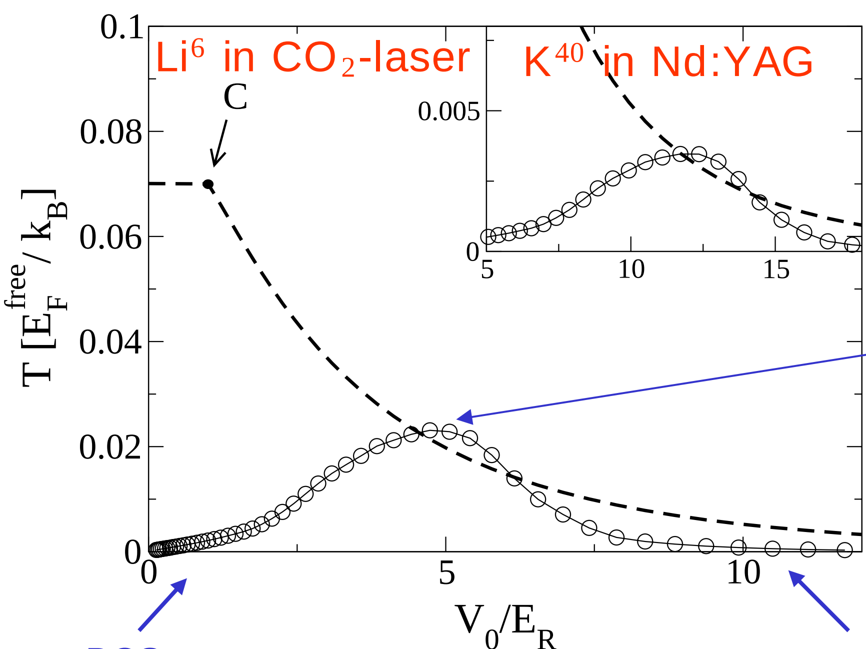
High-T_c superfluidity of fermionic atoms in optical lattice

W. Hofstetter et al., PRL 2003

- Fermi Hubbard model

$$H = -t \sum_{<i,s>,\sigma} (c^{\dagger}_{i\sigma} c_{j\sigma} + h.c.) + u \sum_i n_{i\uparrow} n_{i\downarrow}$$

- example: *u<0* Hubbard model (attractive interactions)



- optical lattice with filling *n~1* $E_F^{\text{free}} \sim E_R \equiv \frac{\hbar^2 k^2}{2m}$

$|u| \sim 10t$

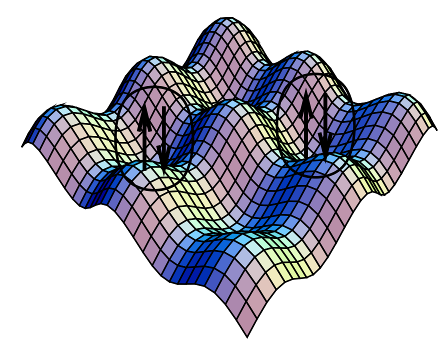
$k_B T_c \sim 0.3 E_F^{\text{free}} k |a_s|$

BCS

$k_B T_c \sim 6t \exp(-\frac{7t}{|u|})$

condensation of preformed boson pairs

$k_B T_c \sim \frac{t^2}{|u|}$



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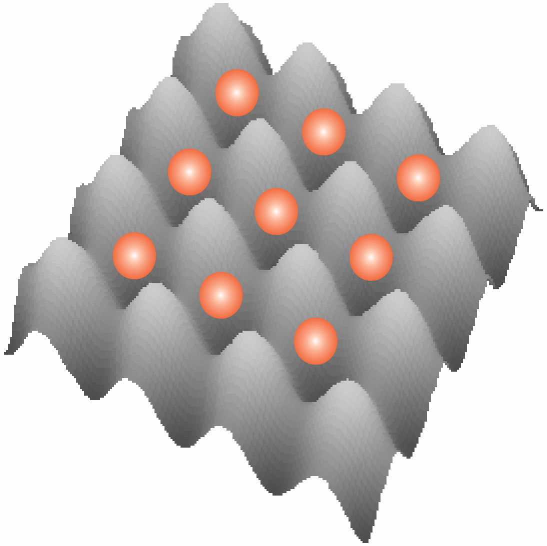
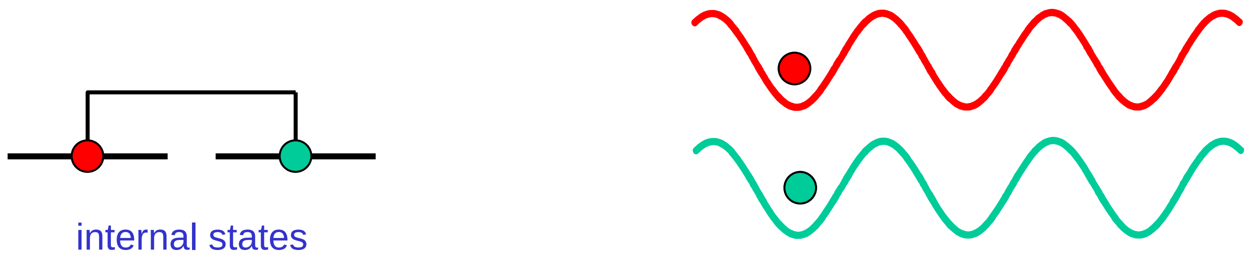


Open questions …

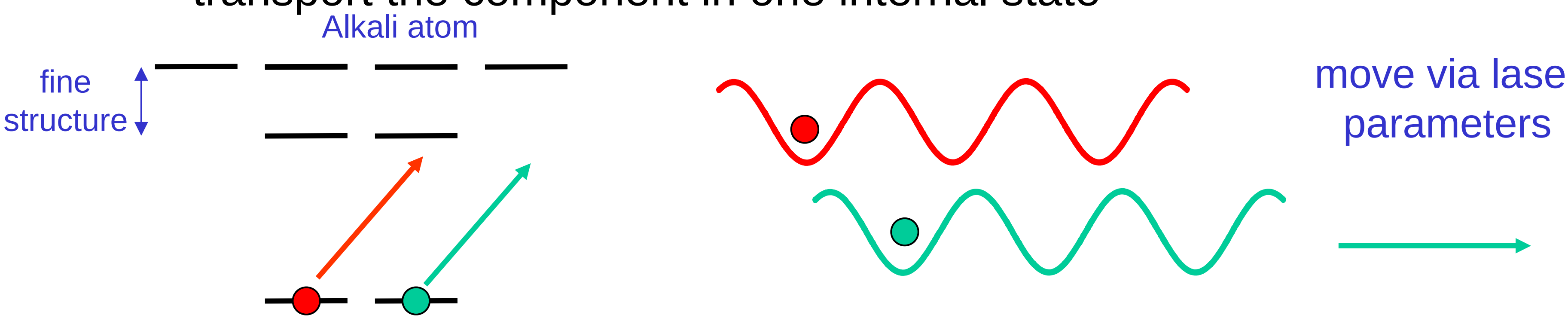
3. entanglement of qubits
4. molecular interactions / physics
5. [phonons]

3. Entanglement by … moving the lattice

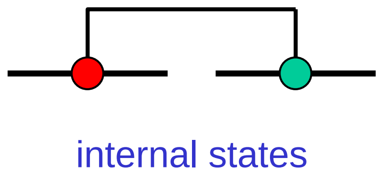
- spin dependent lattices

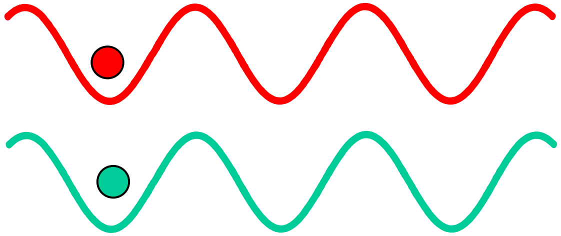


- we can move one potential relative to the other, and thus transport the component in one internal state



- spin dependent lattices





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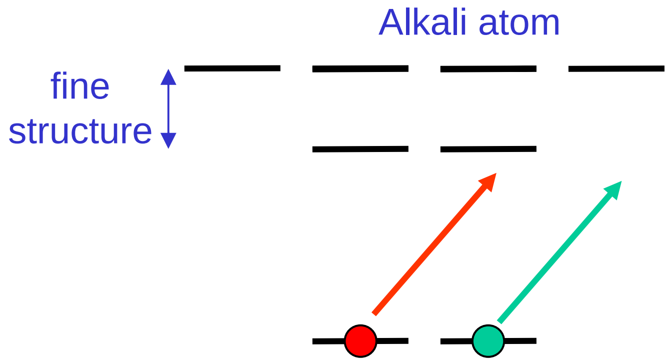
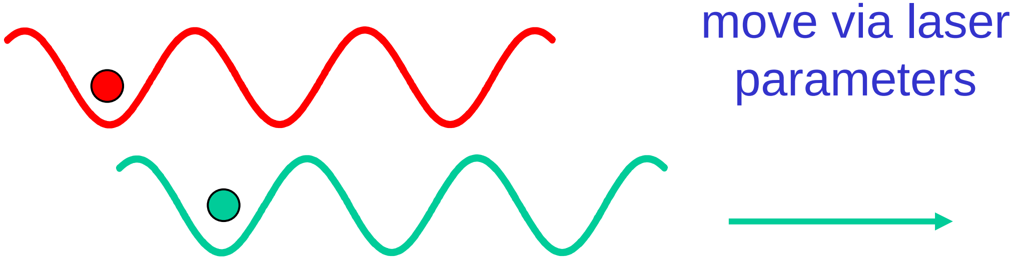
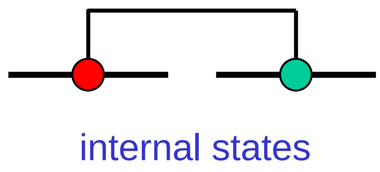
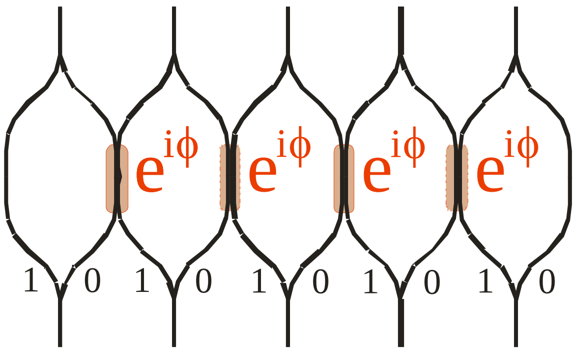
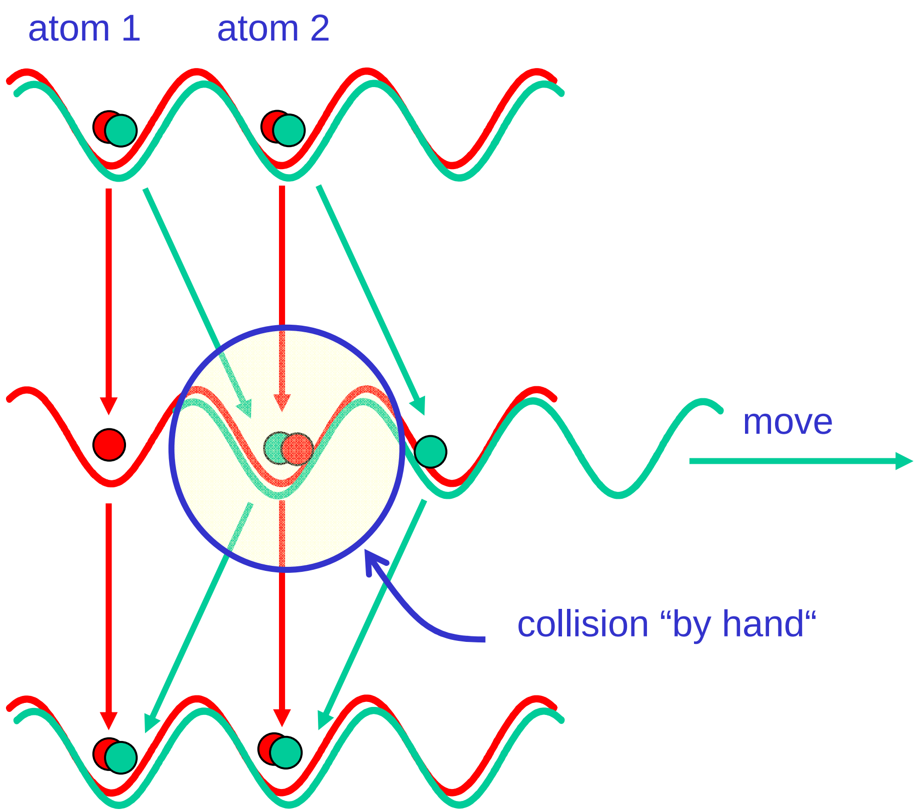




- interactions by moving the lattice + colliding the atoms “by hand”





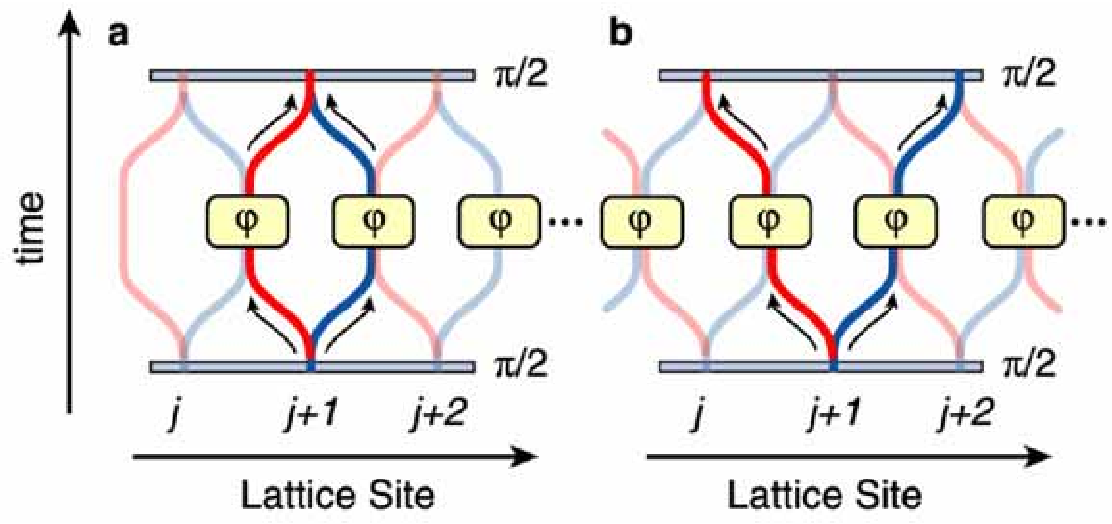


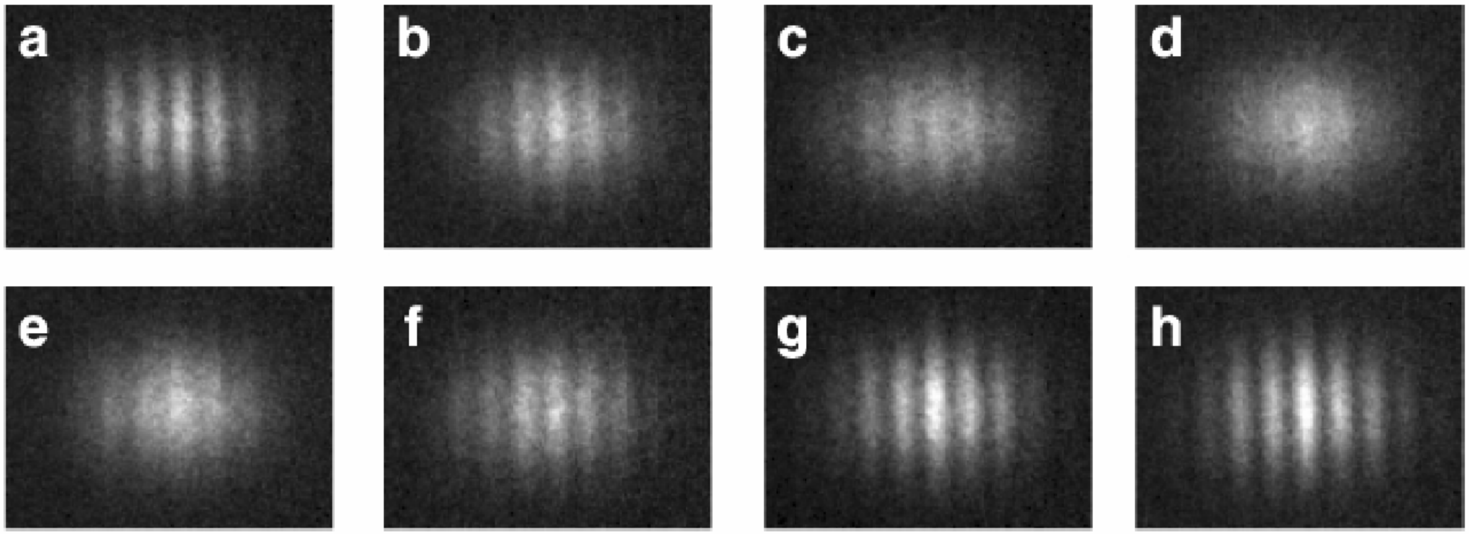
- Ising type interaction (as the building block of the UQS)

$$H = -\frac{J}{2} \sum_{\langle a,b \rangle} \sigma_z^{(a)} \otimes \sigma_z^{(b)}$$

nearest neighbor, next to nearest neighbor ....

Experiment: M. Greiner et al., Nature 2003

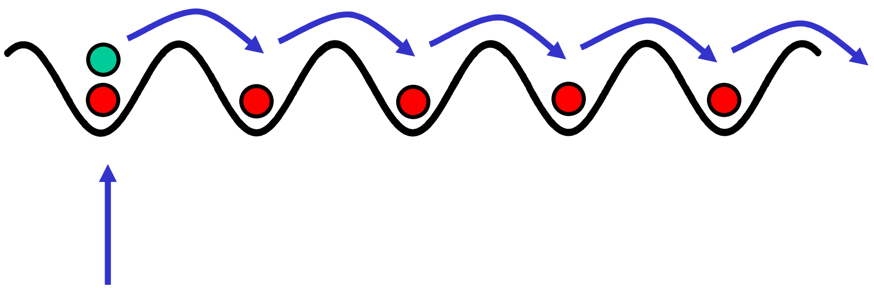




U. Dorner, T. Calarco, P. Julienne, C. Williams, PZ, PRA 2004

Alternative / improved schemes …

- moving a "marker atom" around on the lattice



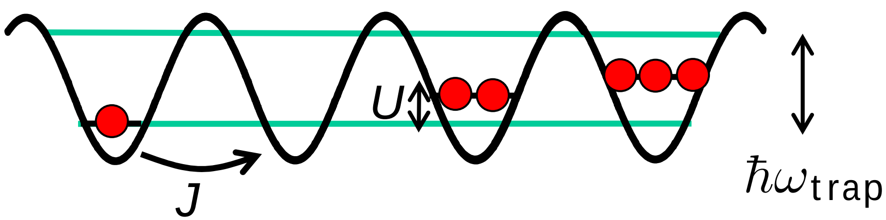
✓ addressing of individual qubits

- magnetic or optical Feshbach resonances

✓ entangling atoms via "marker atoms" as a data bus

4. Hubbard model: microscopic (molecular) picture

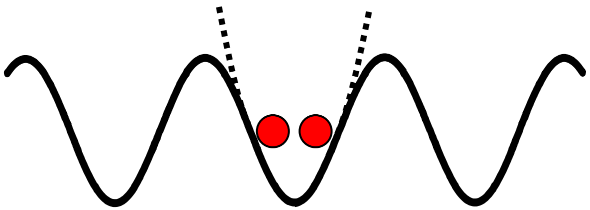
- Hubbard



✓ solve in n=1,2,3,… particle sector

✓ connect by tunneling (e.g in a tight binding approx)

- n=2 atoms on one lattice site: molecule



✓ molecular problem with added optical potential

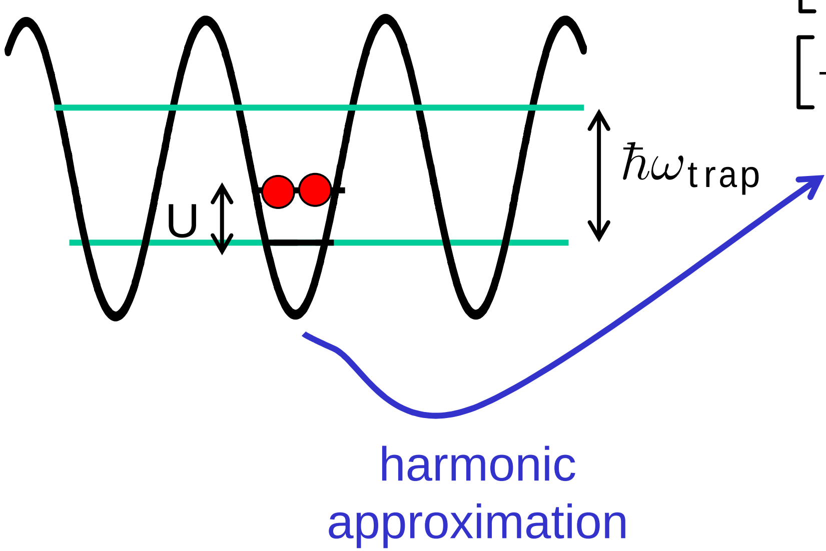
- n=3 atoms on one lattice site: … e.g. Efimov-type problem
- [$n>n_{max}$~3 killed by three body etc. loss]

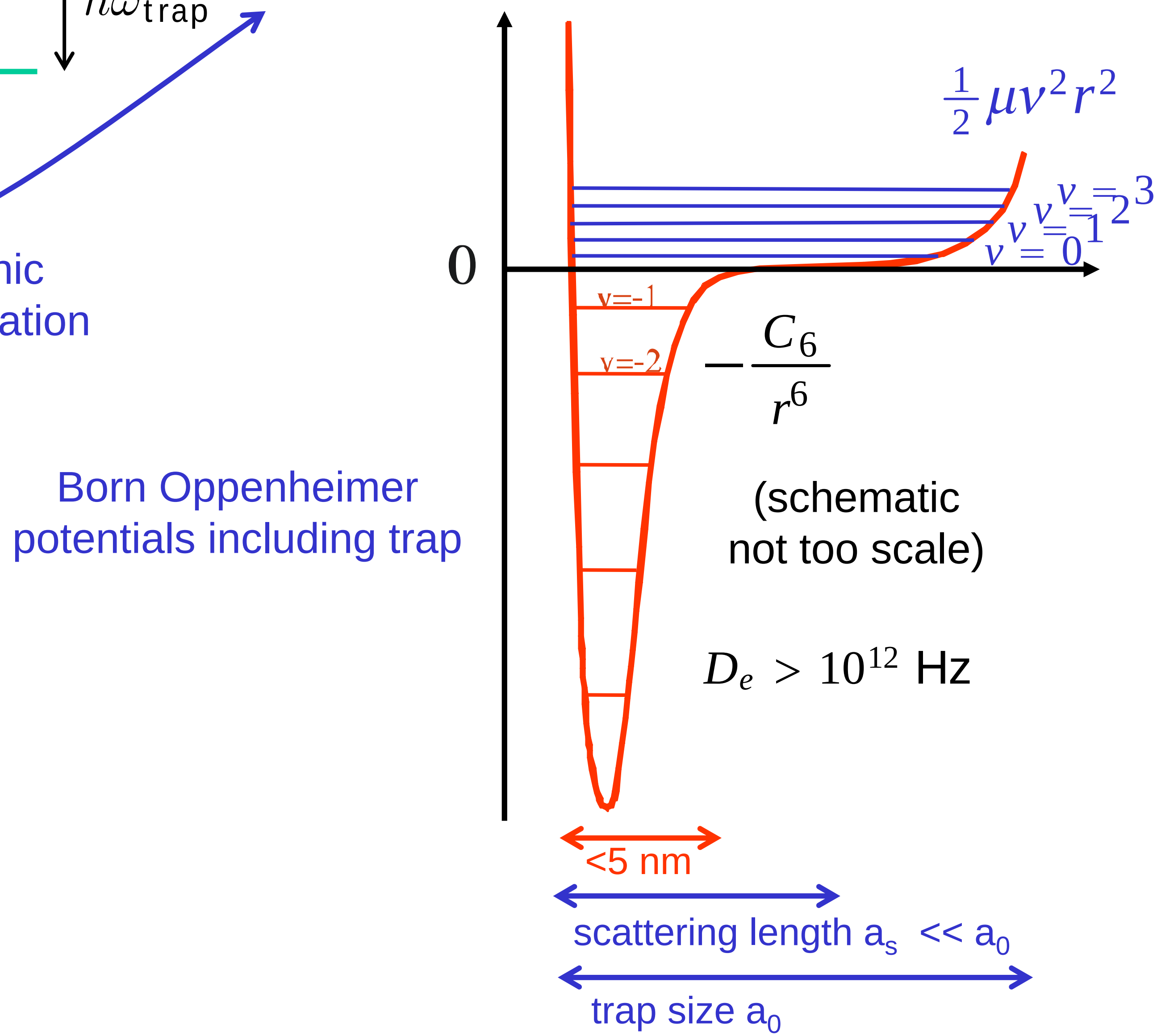
Julienne et al.

- two atoms on one site

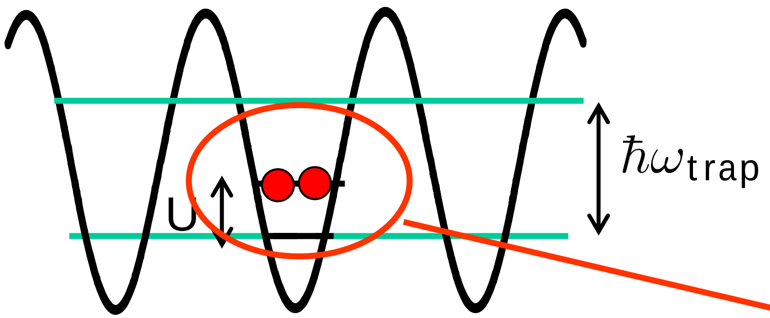
$$\left[-\frac{\hbar^2}{2(2m)}\nabla_R^2 + \frac{1}{2}(2m)\nu^2 R^2\right]\psi_{cm}(R) = E_{cm}\psi_{cm}(R)$$

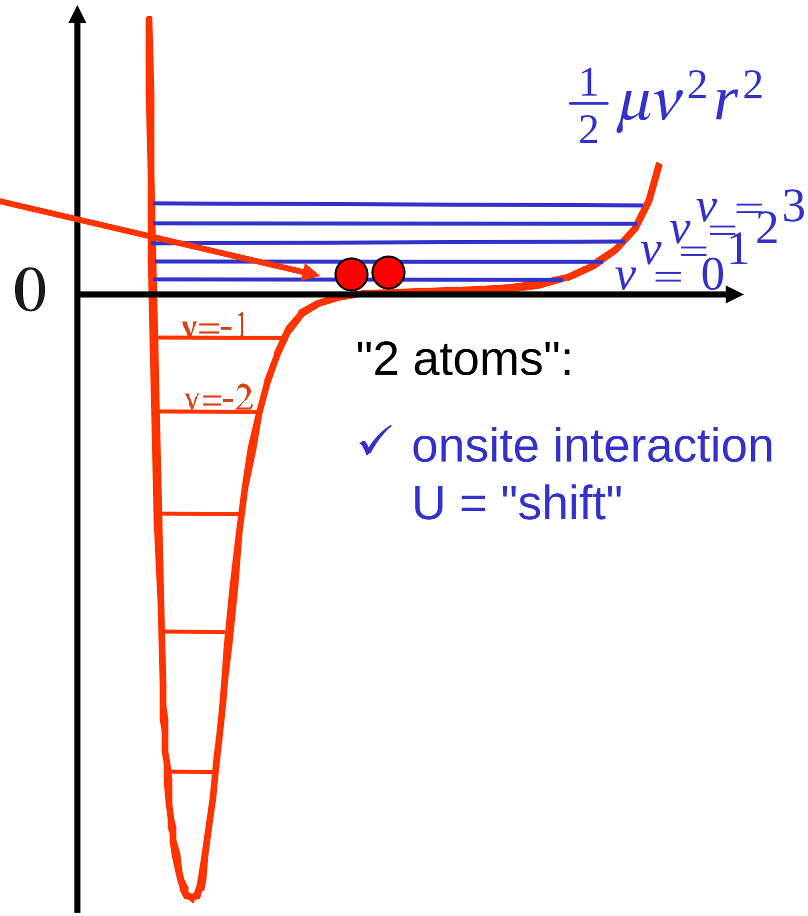
$$\left[-\frac{\hbar^2}{2\mu}\nabla_r^2 + \frac{1}{2}\mu\nu^2 r^2 + V(r)\right]\psi(r) = E\psi(r)$$





- two atoms on one site

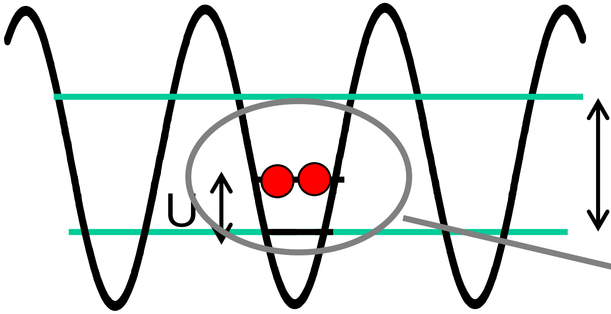


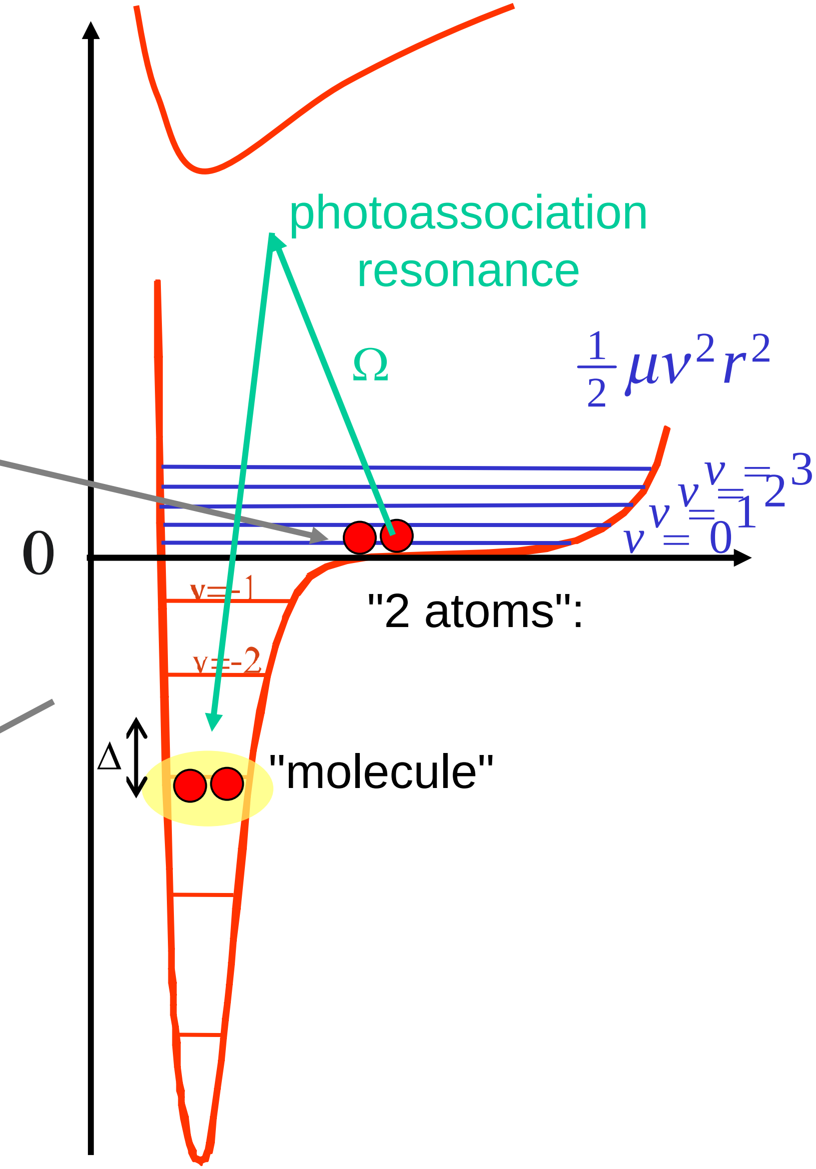


"2 atoms":

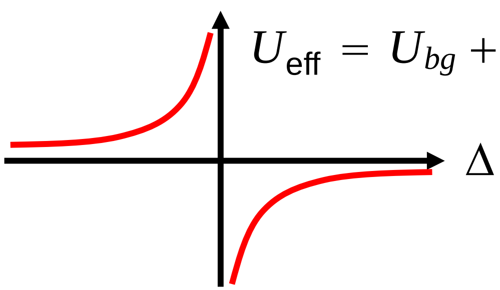
✓ onsite interaction
U = "shift"

- two atoms on one site





AC Starkshift = optical Feshbach resonance

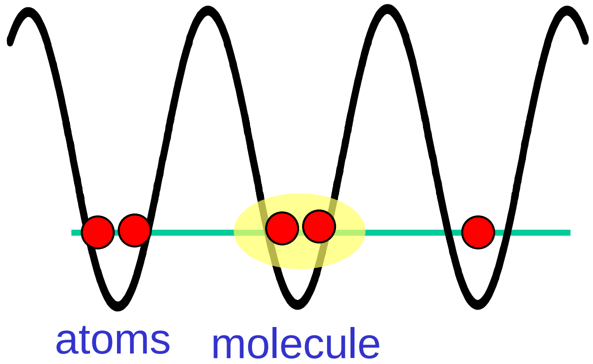


$$U_{\text{eff}} = U_{bg} + \frac{1}{4}\frac{\Omega^2}{\Delta}$$

$$H = (U_{bg} + \frac{1}{4}\frac{\Omega^2}{\Delta})b^{\dagger 2}b^2$$

Hubbard model including molecules

- Hamiltonian



$$H = -J_b \sum_{<i,j>} b_i^\dagger b_j + \frac{1}{2} U_b \sum_i b_i^\dagger b_i^\dagger b_i b_i$$
$$- J_m \sum_{<i,j>} m_i^\dagger m_j + \frac{1}{2} U_m \sum_i m_i^\dagger m_i^\dagger m_i m_i - \sum_i \Delta m_i^\dagger m_i$$
$$+ \frac{1}{2} \Omega \sum_i m_i^\dagger b_i b_i + \text{h.c.}$$

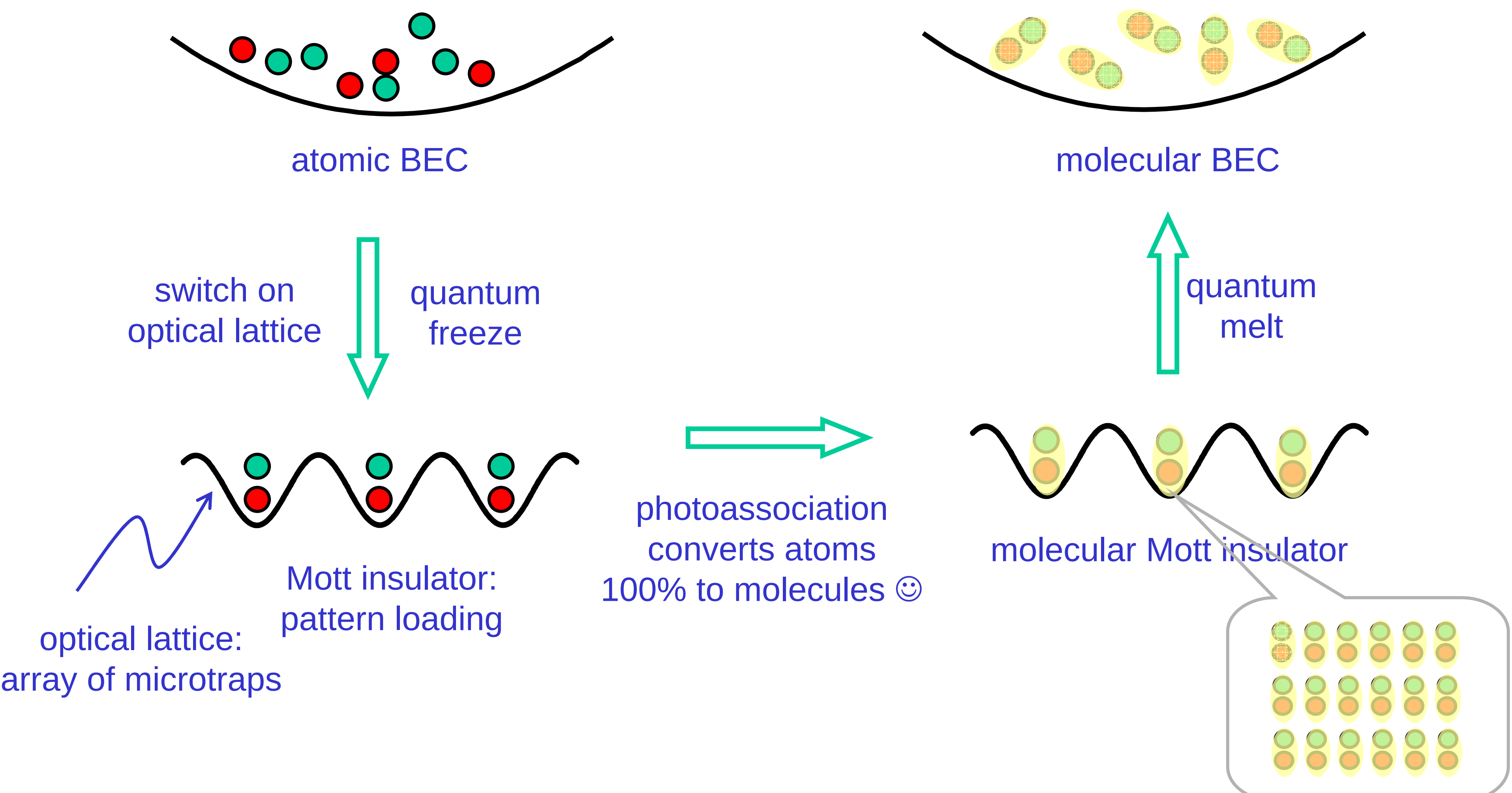
Remarks:

✓ we have derived this only for sector: 2 atoms or 1 molecule

✓ inelastic collisions / loss for >2 atoms and >1 molecules (?)

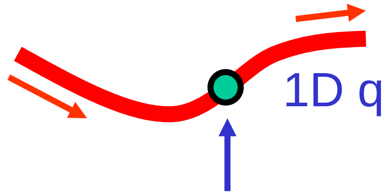
Remark: quantum phases of "composite objects"

- molecular BEC via a quantum phase transition



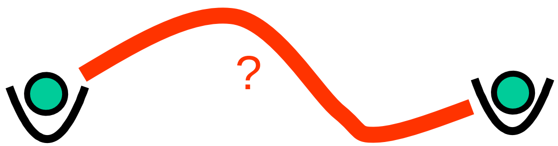
Present research topics

- **Single Atom Transistor** with 1D optical lattices



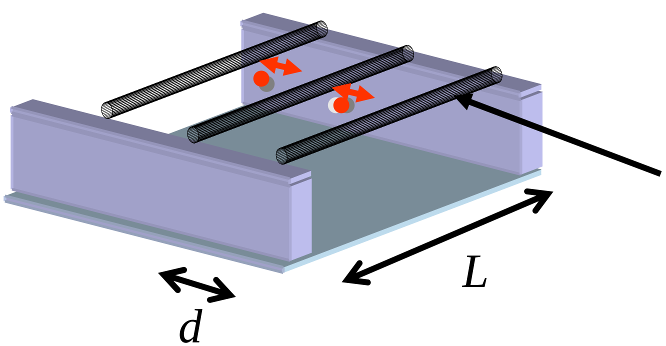
A. Micheli, A.Daley,
D. Jaksch & PZ,
PRL in press

- **Interfacing quantum optical & mesoscopic condensed matter systems**



connecting two quantum optical qubits by a
(passive / active) *solid state* bus

Lin Tian, PZ,
PRL 2004 and submitted



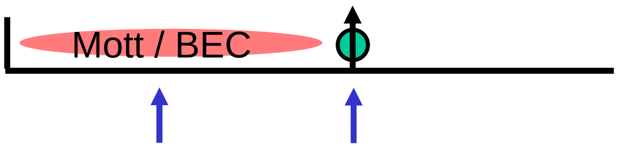
nanotubes:

✓ ion trap electrodes

✓ nanomechanical oscillator

Atomic Quantum Switch: motivation & background

- "single atom quantum" mirrors for 1D quantum gas



bosonic atoms

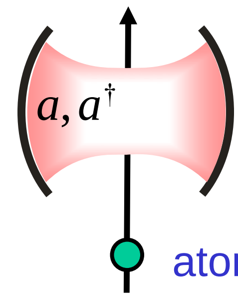
✓ interacting

single atom as a …

✓ mirror

✓ qubit = spin-1/2

- Cavities and CQED



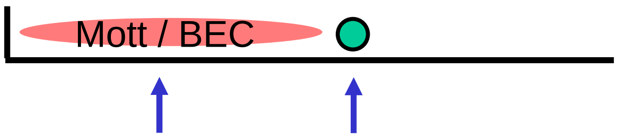
cavity mode: bosonic

atom: spin-1/2

- Questions …
  - how?
  - why?
  - (time dependent) dynamics?
  - [measurement]

- Jaynes-Cummings model / quantum state engineering
- dissipative / open quantum system / measurement: e.g.QND
- experimental realization:
  - microwave (Haroche, …)
  - optical (Kimble, Rempe, Feld, …)

- "single atom quantum" mirrors for 1D quantum gas



bosonic atoms

✓interacting

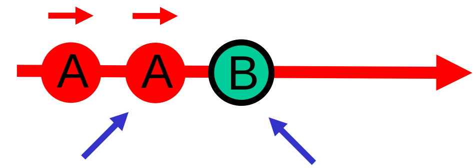
✓1D condensate
… Tonks gas

single atom as a …

✓ mirror

✓ qubit = spin-1/2

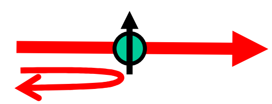
- impurity atom in a 1D atomic wire



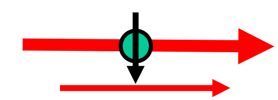
A atoms as
atomic current

fixed B atom:
impurity

- qubit B: spin



or

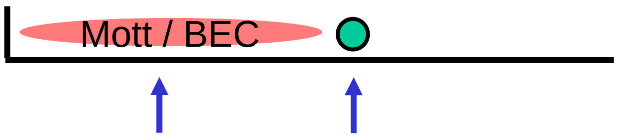


spin up:
reflection

spin down:
transmission

"spintronics"

- "single atom quantum" mirrors for 1D quantum gas



bosonic atoms

✓interacting

✓1D condensate
… Tonks gas

single atom as a …

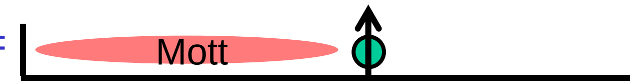
✓ mirror

✓ qubit = spin-1/2

- qubit B: spin

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle$$

qubit “0” = BLOCK:



qubit “1” = OPEN:

$$\alpha|\uparrow\rangle|\text{Mott}\rangle + \beta|\downarrow\rangle|\text{BEC}\rangle$$

✓"amplify" spin

✓ entangled quantum phases

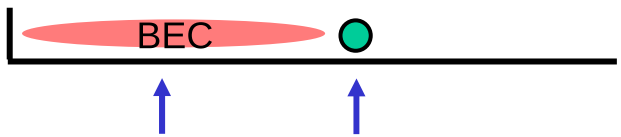
- Quantum non-demolition measurement: qubit read out

$$\alpha|\uparrow\rangle|\text{Mott}\rangle + \beta|\downarrow\rangle|\text{BEC}\rangle$$

no interference

interference

- "single atom transitor"



bosonic atoms

✓interacting

✓1D condensate
… Tonks gas

single atom as a …

✓ mirror

✓ qubit = spin-1/2

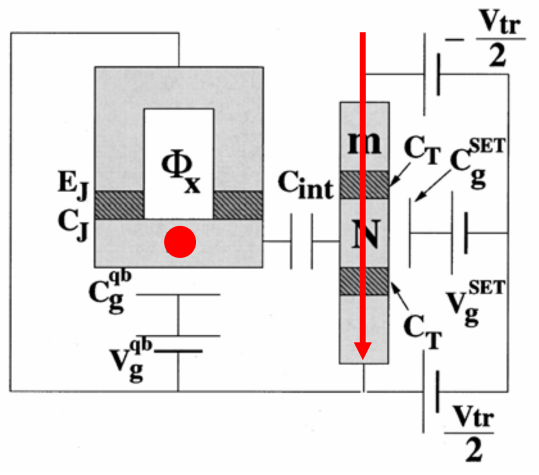
- qubit B: spin

spin up:
reflection

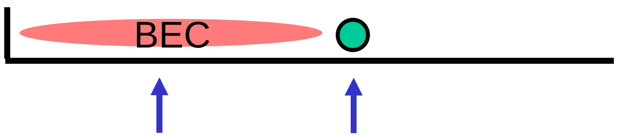
spin down:
transmission

coherent "spintronics"

- reminiscent of single electron transistor



- "single atom quantum" mirrors for 1D quantum gas



bosonic atoms

✓interacting

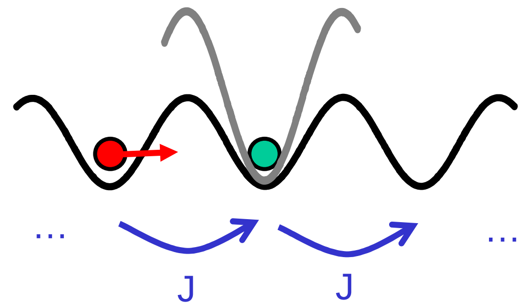
✓1D condensate
… Tonks gas

single atom as a …

✓ mirror

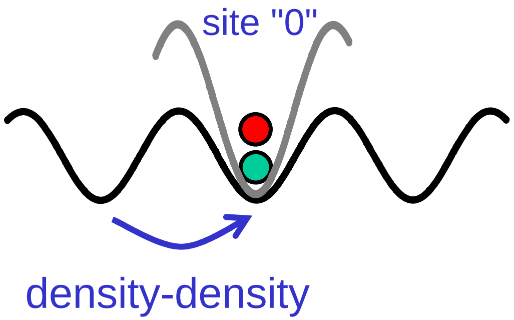
✓ qubit = spin-1/2

- realization: 1D optical lattice



How?

- collisional interactions

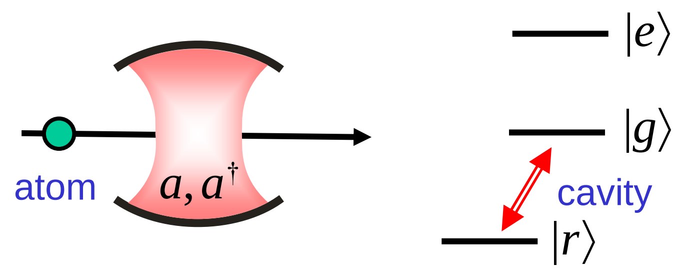


$$H = U_{ab} a_0^\dagger a_0 b_0^\dagger b_0$$

+ spin

$$H \sim a_0^\dagger a_0 \hat{\sigma}_z$$

- compare: Cavity QED



$$H \sim a^\dagger a |g\rangle\langle g| \sim a^\dagger a \hat{\sigma}_z$$

AC-Starkshift

$$[a^\dagger a, H] = 0$$

QND photon number

- **goal:** large interactions
- **answer:** Feshbach or photo association resonances
  - large scattering length
  - **better: infinite scattering length**

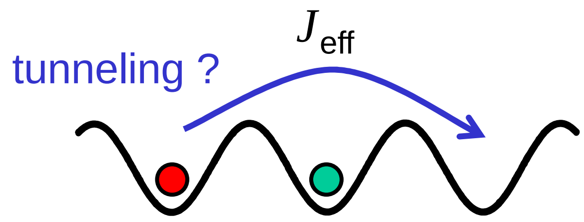
new physics:

- EIT-type quantum interference to kill transport

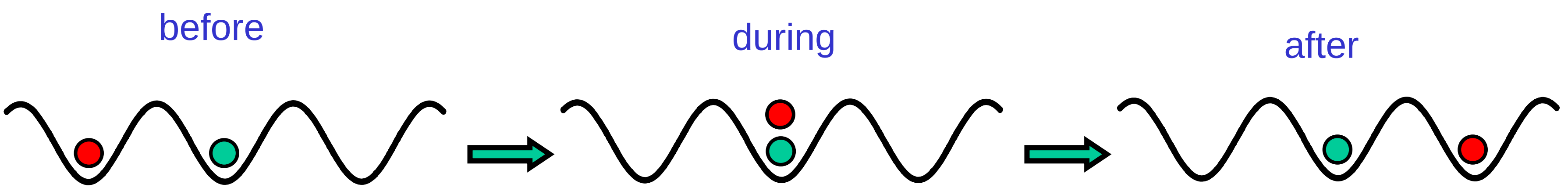
Atomic Switch by Quantum Interference

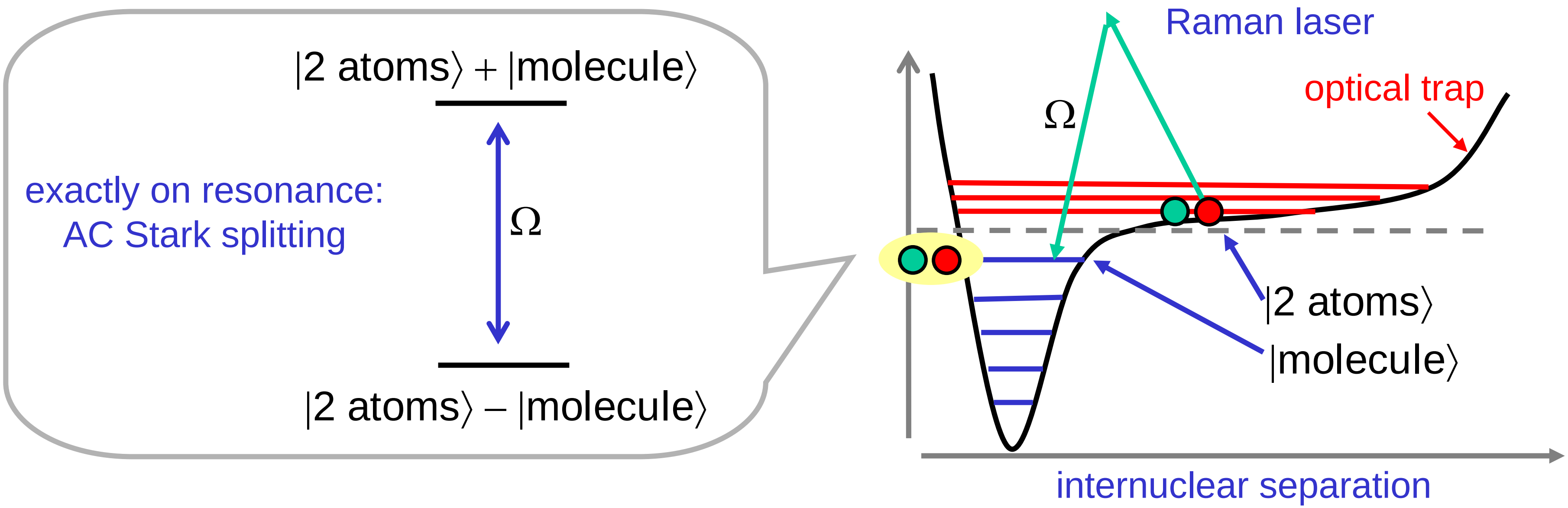
Micheli et al.

- tunneling through the quantum dot?

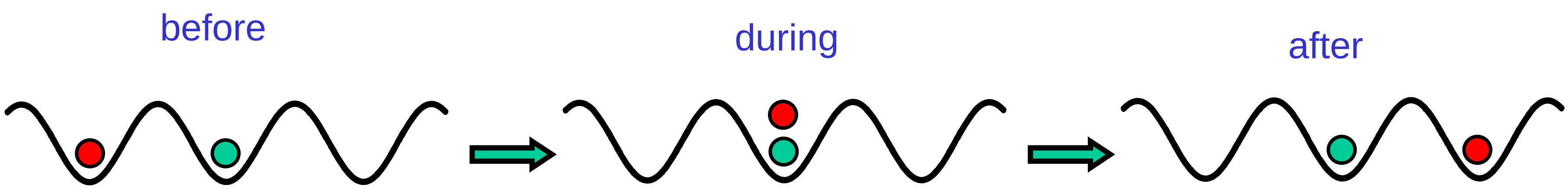


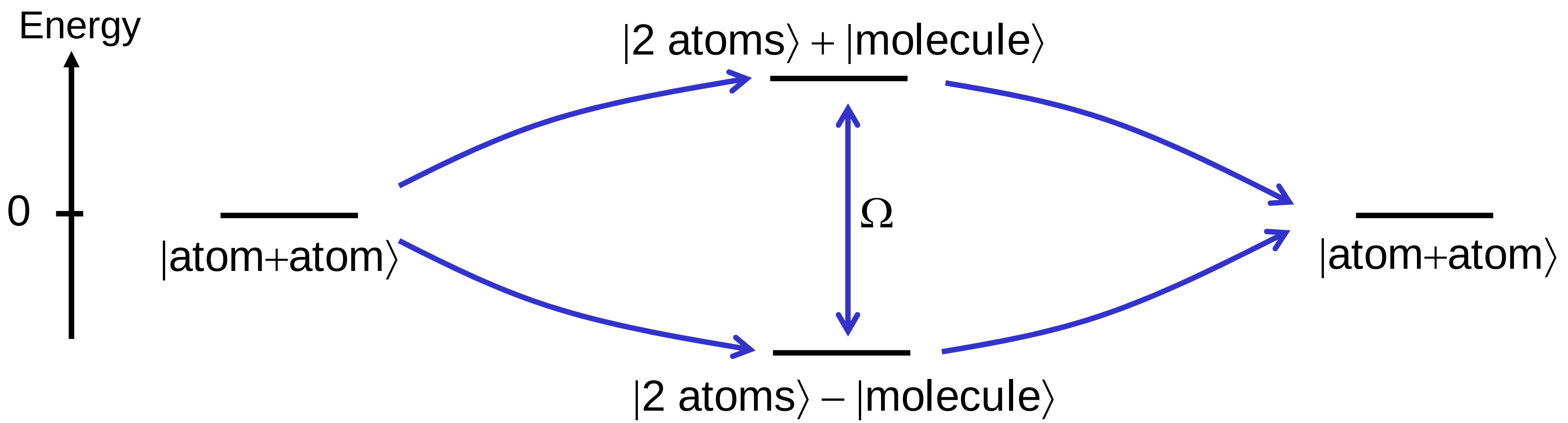
- process in an optical lattice





- blocking by quantum interference





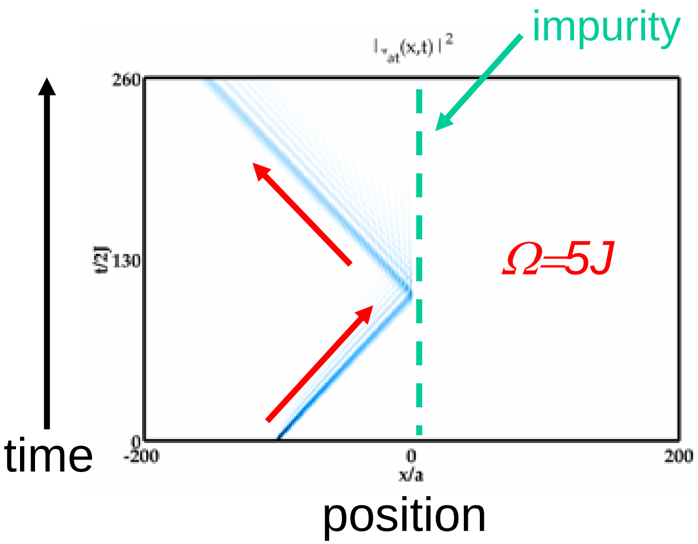
$$J_{\mathrm{eff}} \sim J \frac{1}{E-\frac{1}{2}\Omega} J + J \frac{1}{E+\frac{1}{2}\Omega} J \overset{!}{=} 0 \quad \text{(for } E = 0\text{)}$$

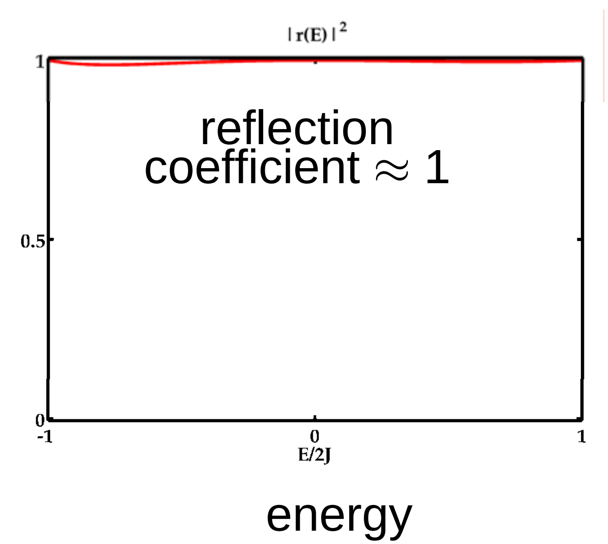
we kill the transport by quantum interference:
"infinite repulsion" (EIT)

… testing these ideas for *single* atoms

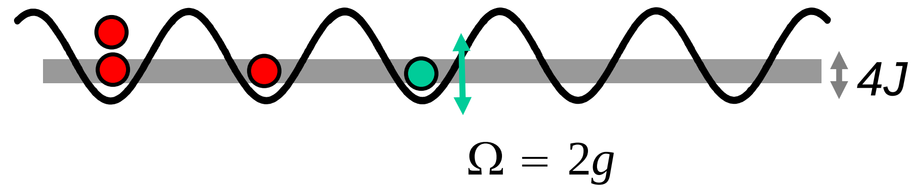
- we can solve the scattering problem of one atom from the impurity exactly (Lippmann Schwinger equation)

- numerical solution of the Schrödinger equation: wave packet dynamics in the limit of strong PA laser





Many A atoms



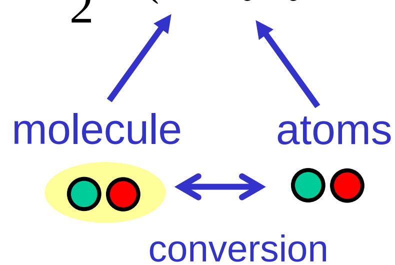
- Hubbard model with impurity on site "0"

$$H = -\sum_i J(a_i^\dagger a_{i+1} + a_{i+1}^\dagger a_i) + \frac{1}{2} U \sum_i a_i^{\dagger 2} a_i^2$$

dynamics of atoms

$$+ U_{bg} a_0^\dagger a_0 b^\dagger b - \Delta m^\dagger m + \frac{1}{2}\Omega(m^\dagger a_0 b_0 + a^\dagger b^\dagger m)$$

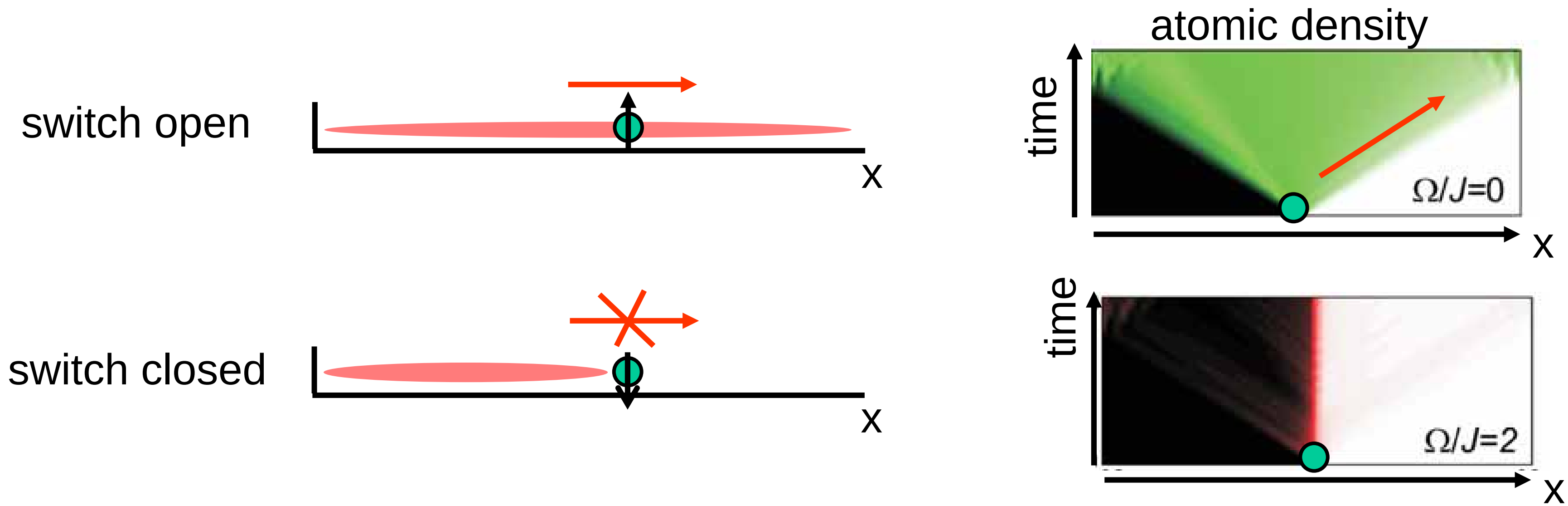
impurity dynamics



- [Exact] Solution of *time dependent* many body Schrödinger equation for up to ~ 100 atoms
  – numerical: time dep DMRG-type method G. Vidal, PRL 91, 147902 (2003)
  – semianalytical: hard core bose gas of atoms

Time dependent many body dynamics: results

- numerical results for N ~ 30 atoms on 61 lattice sites

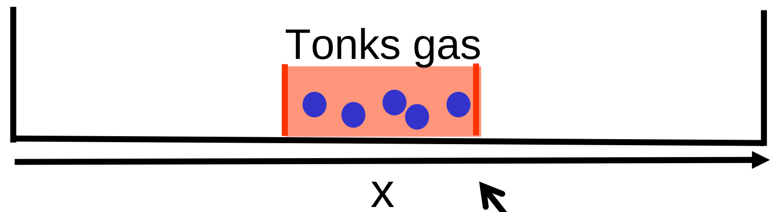


- current-voltage characteristics for transport …

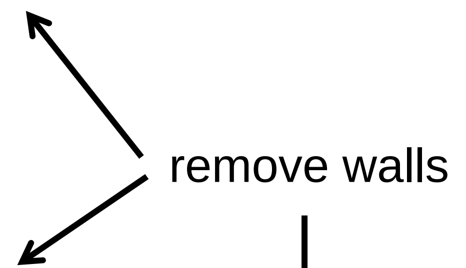
- works great!

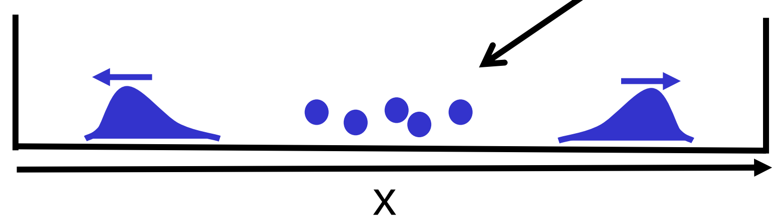
Tonks gas: release & time evolution

- prepare a Tonks gas (hard core bosons)



- time evolution after release





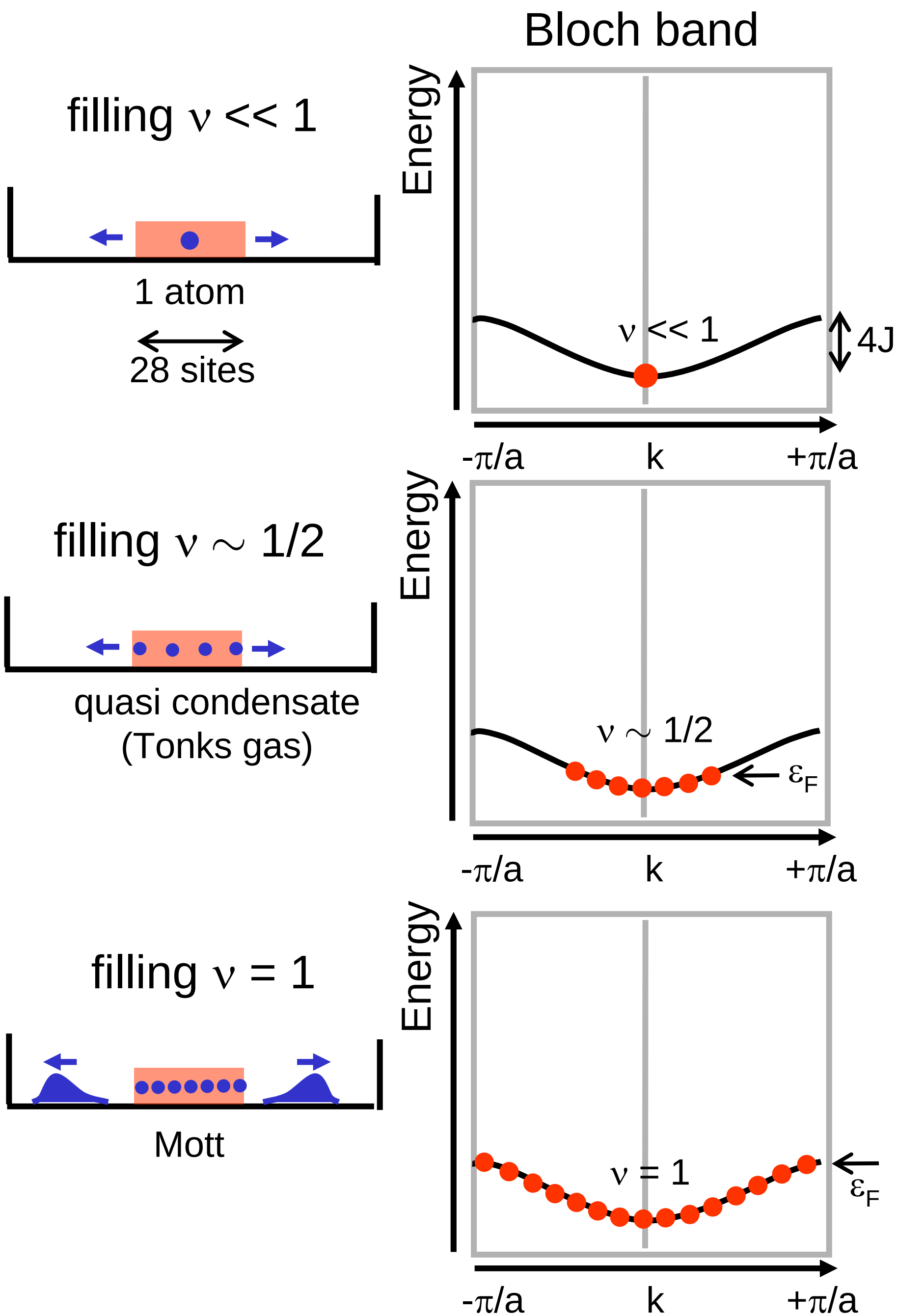
- Hubbard

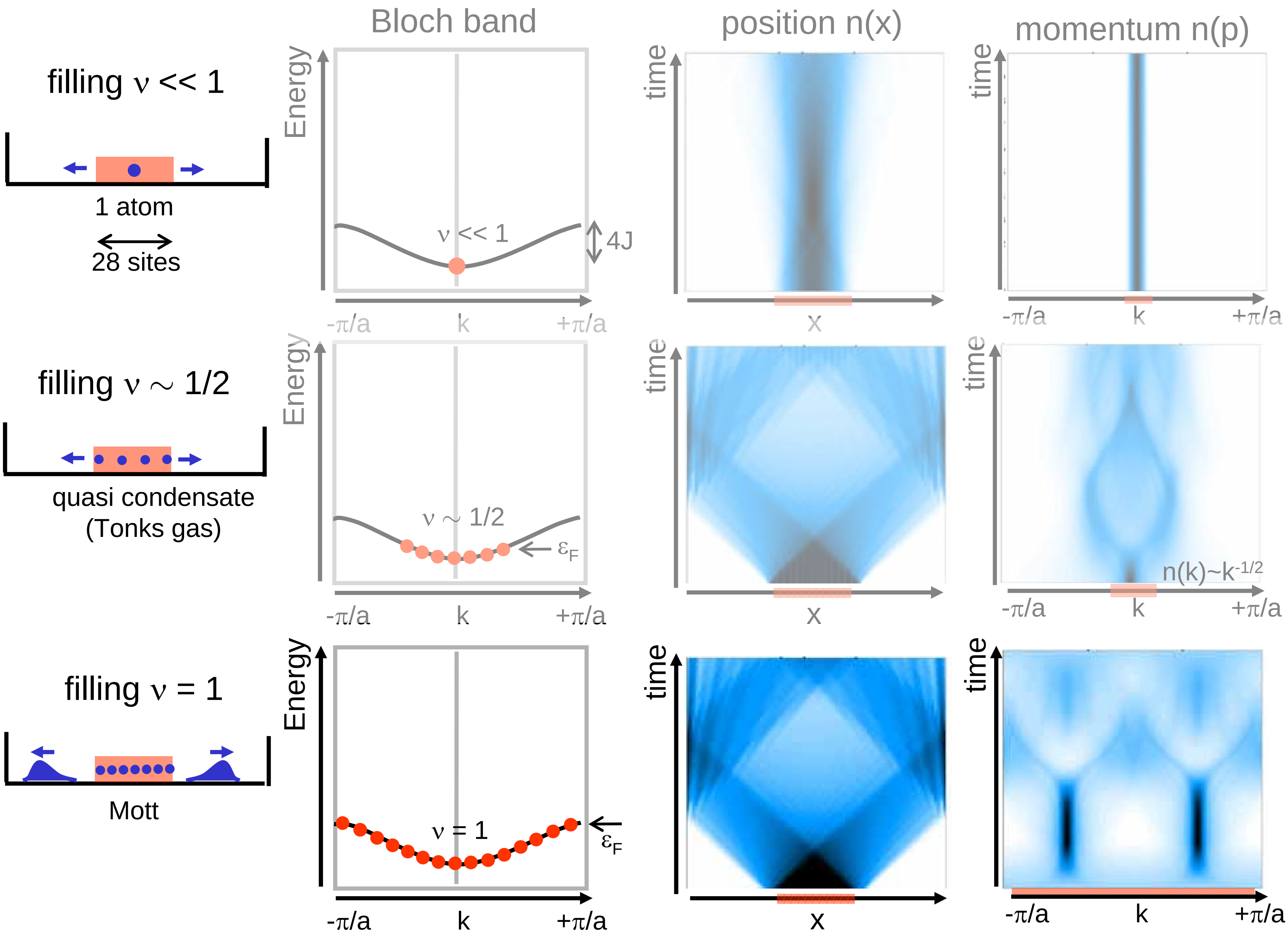
$$H_{\mathrm{B}} = -J \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum n_j (n_j - 1) \qquad [b_i, b_j^\dagger] = \delta_{ij}$$

trivial fermionization

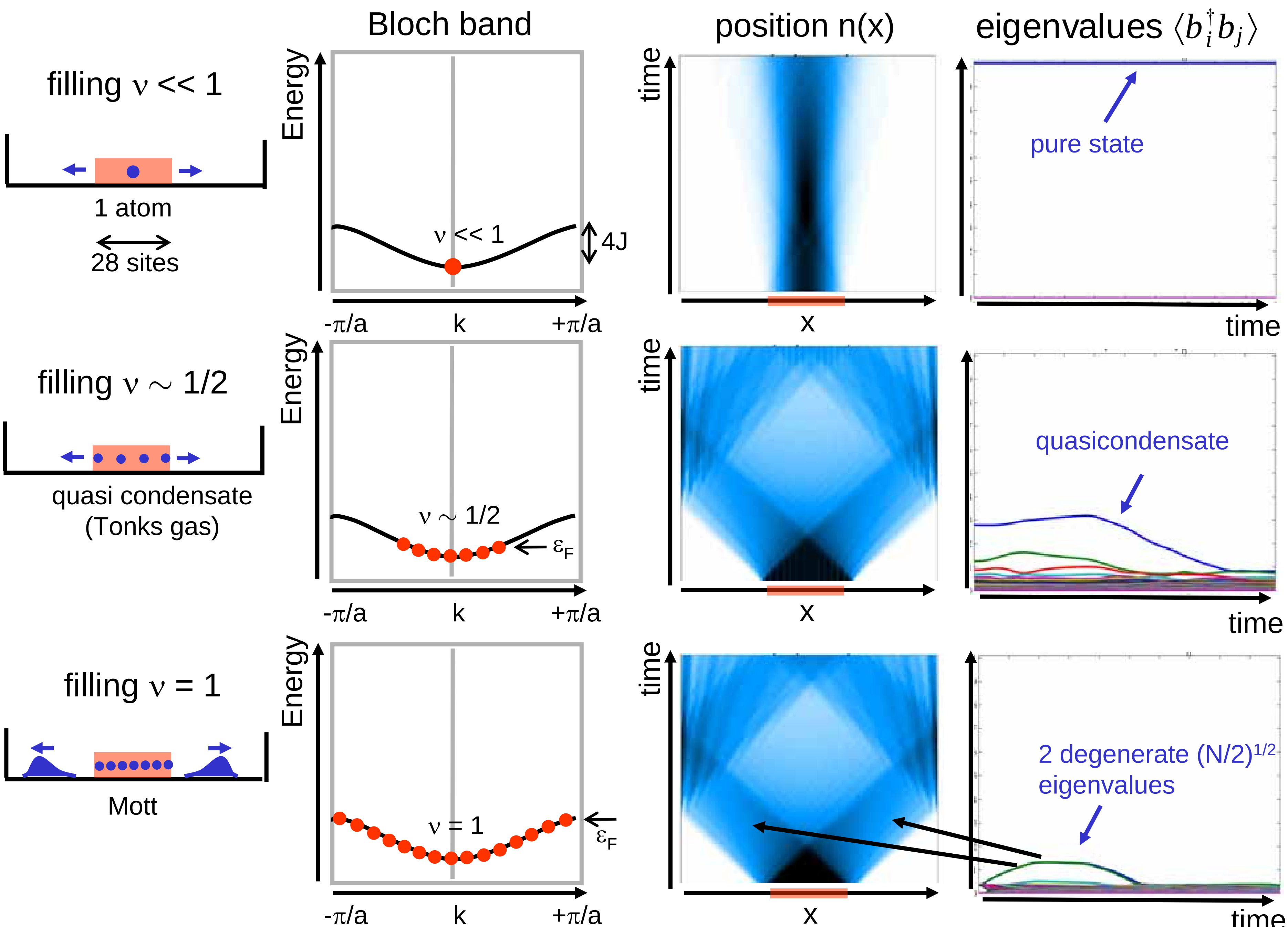
$$c_j = \sigma_j \prod_{l<j} (-1)^{n_l}$$

$$H_{\mathrm{F}} = -J \sum_{j=-M}^{M-1} c_j^\dagger c_{j+1} + \mathrm{h.c.}$$


Bloch band
filling ν << 1
1 atom
28 sites
Energy
ν << 1
4J
-π/a
k
+π/a
filling ν ~ 1/2
quasi condensate
(Tonks gas)
Energy
ν ~ 1/2
ε_F
-π/a
k
+π/a
filling ν = 1
Mott
Energy
ν = 1
ε_F
-π/a
k
+π/a



Bloch band
position n(x)
momentum n(p)
filling ν << 1
1 atom
28 sites
Energy
ν << 1
4J
time
-π/a
k
+π/a
x
filling ν ~ 1/2
quasi condensate
(Tonks gas)
ν ~ 1/2
εF
n(k)~k-1/2
filling ν = 1
Mott
ν = 1



Bloch band
position n(x)
eigenvalues $\langle b_i^\dagger b_j \rangle$
filling ν << 1
1 atom
28 sites
Energy
ν << 1
4J
-π/a
k
+π/a
time
x
pure state
time
filling ν ~ 1/2
quasi condensate
(Tonks gas)
ν ~ 1/2
ε_F
quasicondensate
filling ν = 1
Mott
ν = 1
2 degenerate (N/2)^1/2
eigenvalues

Conclusion

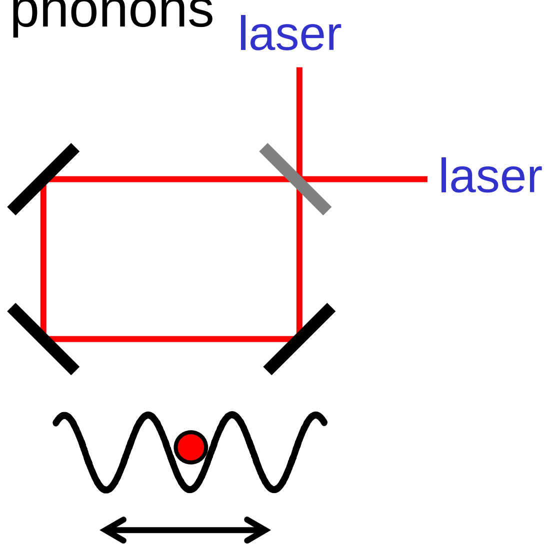
- **Cold atoms in optical lattices**
  - toolbox for engineering of condensed matter model Hamiltonians
  - quantum simulator for exotic quantum phases / excitations
  - entangled state engineering
    - quantum computing
    - special purpose entangled states: squeezed states (for application in metrology)

- **Single atom transistor**
  - towards atomic circuits

Jaksch, Griessner,
Daley, PZ

Phonons

- cavity mode dynamics as phonons



✓ standing wave responds to the atomic motion

✓ single mode (!)

- laser assisted phonon cooling
  - in analogy to laser cooling with photon → phonon