Cosmic problems for condensed matter experiment

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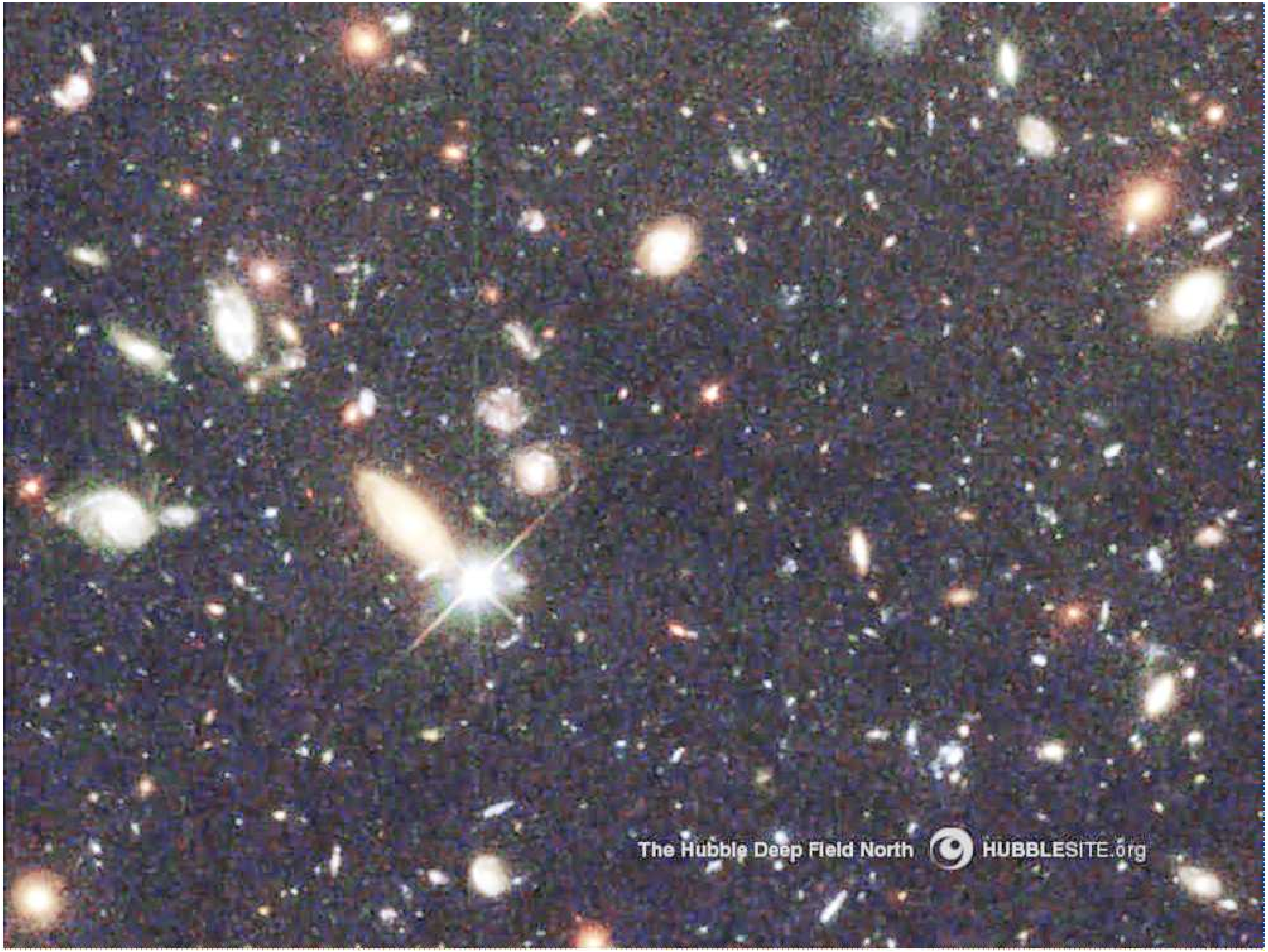
State of the universe



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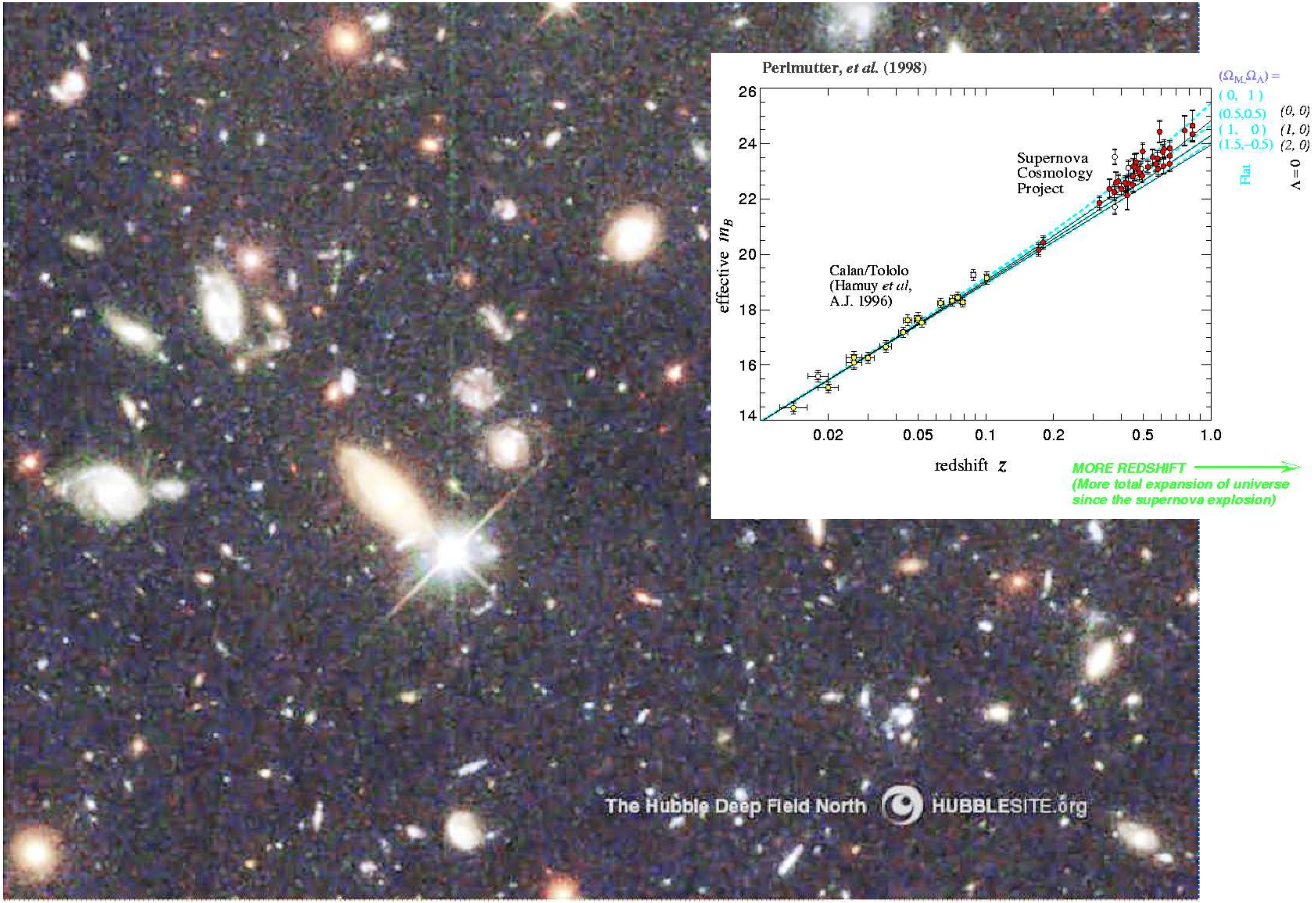

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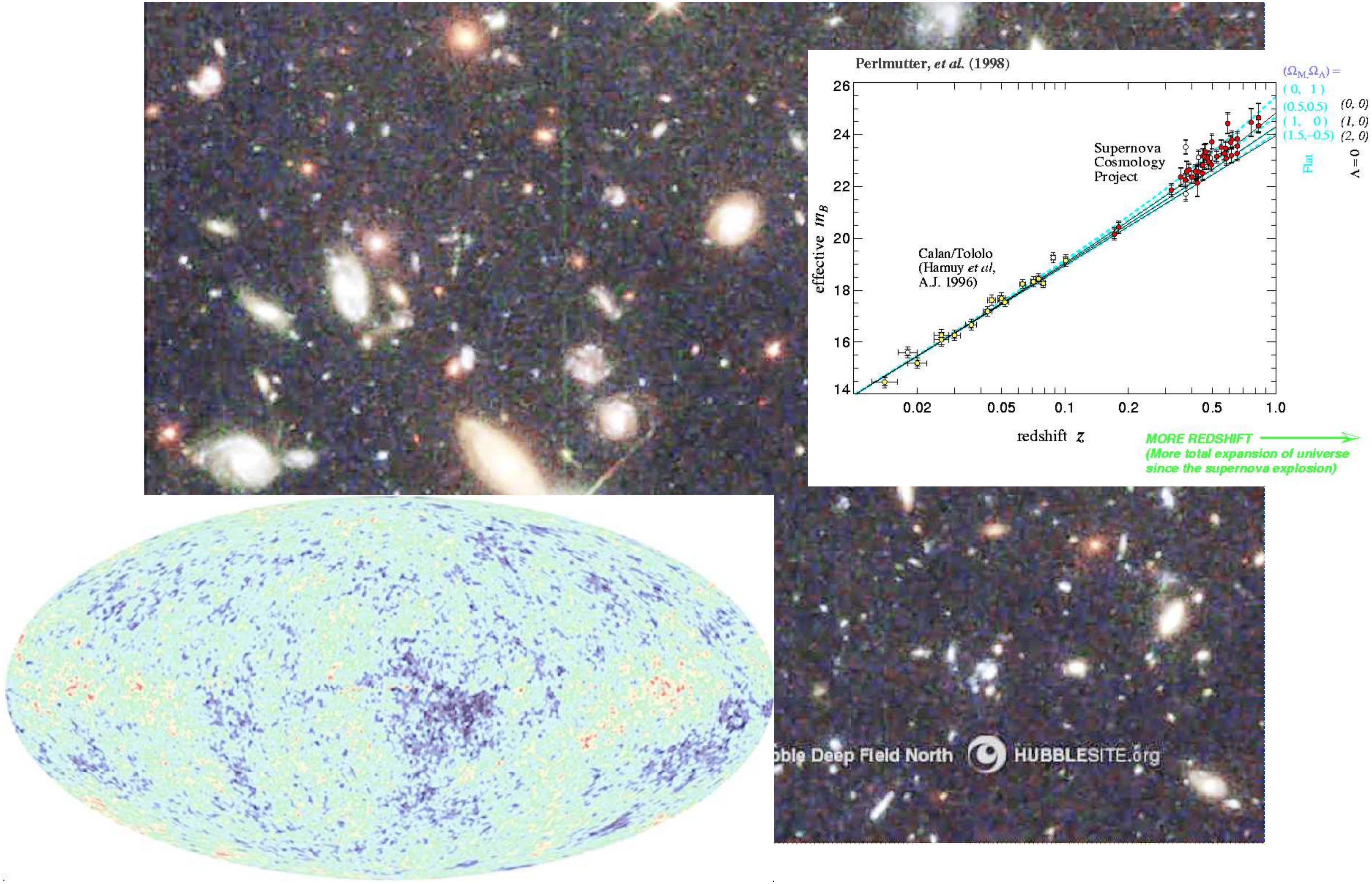

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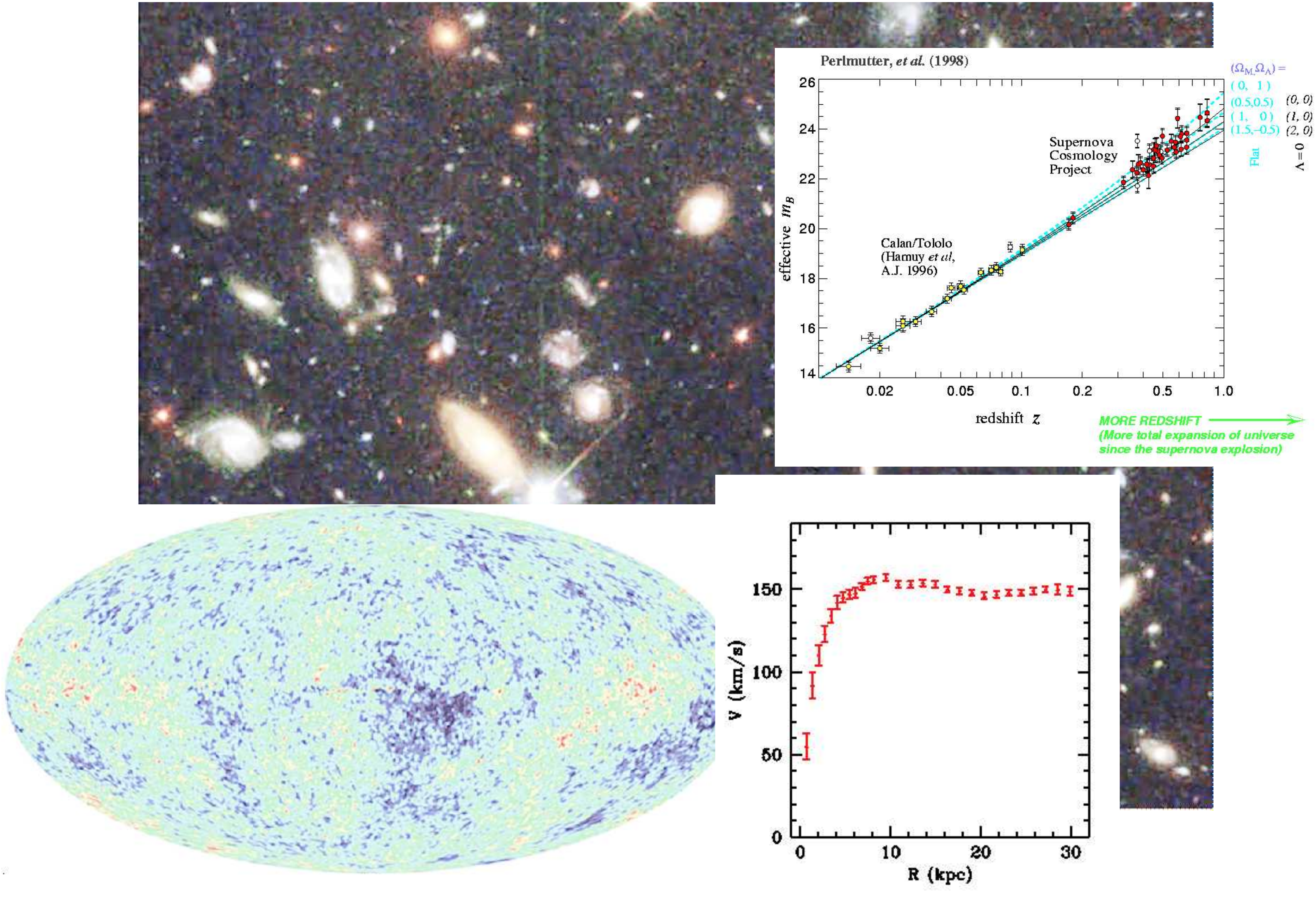

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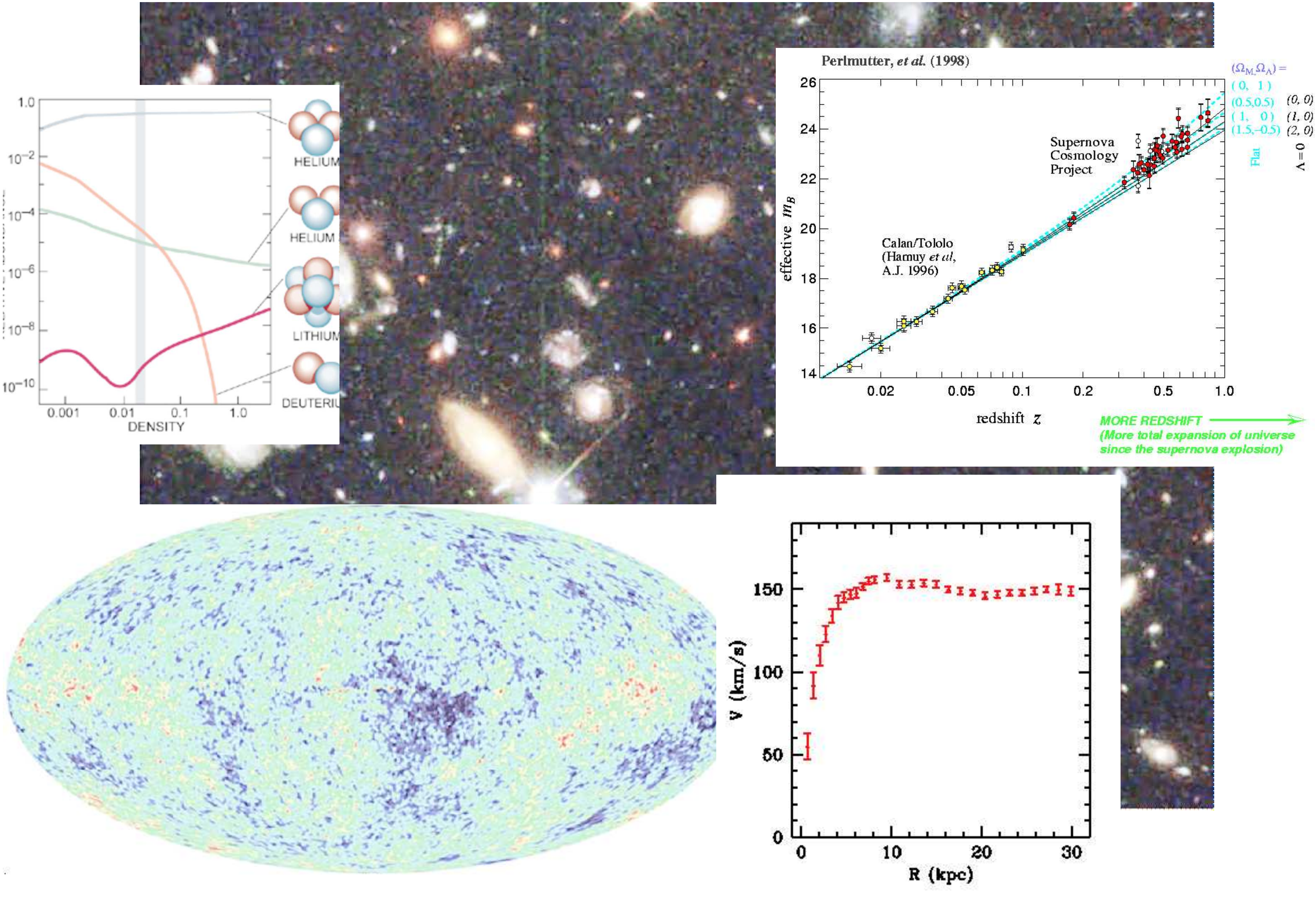
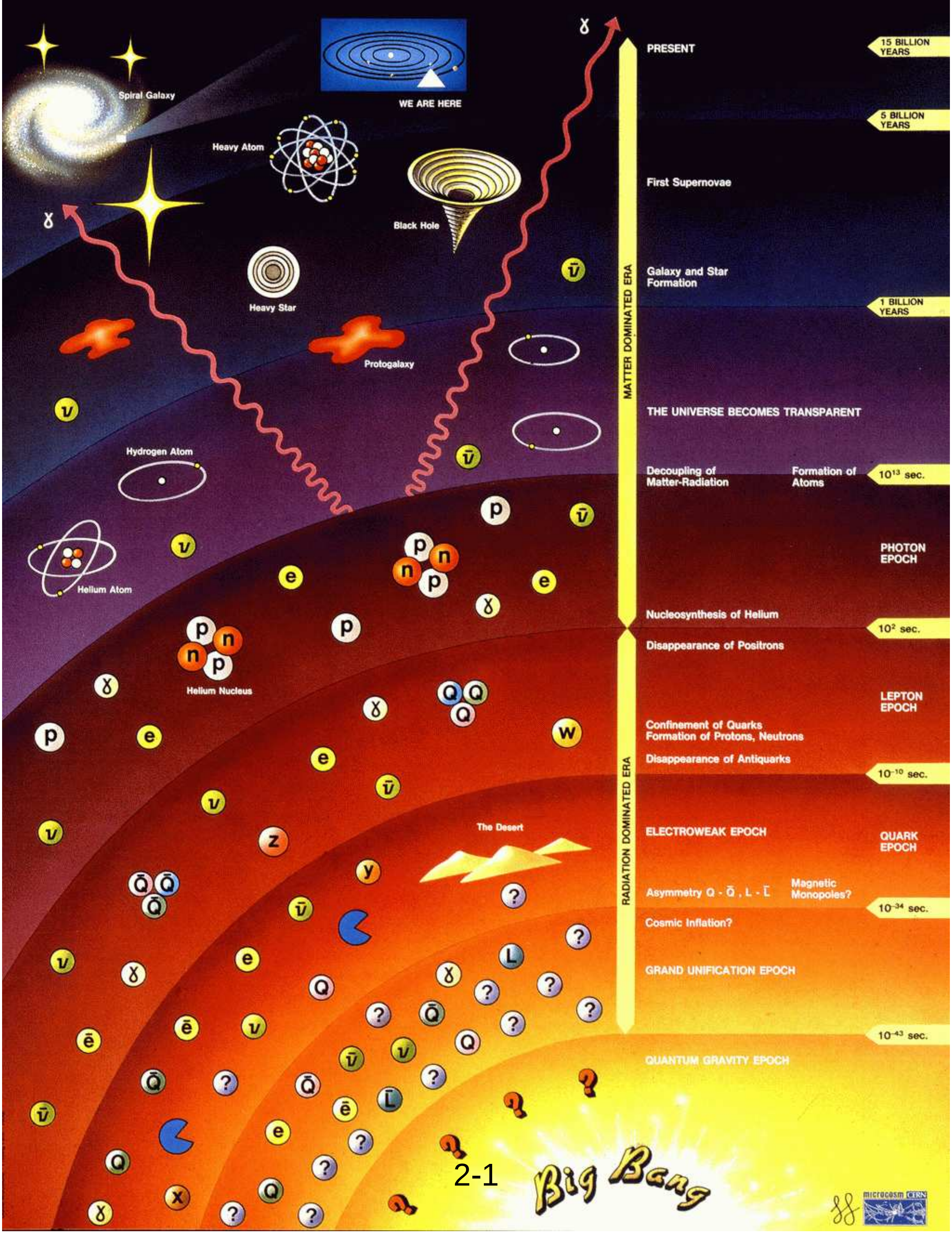


History of the Universe



MATTER DOMINATED ERA

PRESENT — 15 BILLION YEARS

5 BILLION YEARS

First Supernovae

Galaxy and Star Formation

1 BILLION YEARS

THE UNIVERSE BECOMES TRANSPARENT

Decoupling of Matter-Radiation — Formation of Atoms — 10^{13} sec.

PHOTON EPOCH

Nucleosynthesis of Helium

10^{2} sec.

RADIATION DOMINATED ERA

Disappearance of Positrons

LEPTON EPOCH

Confinement of Quarks
Formation of Protons, Neutrons

Disappearance of Antiquarks

10^{-10} sec.

ELECTROWEAK EPOCH

QUARK EPOCH

Asymmetry $Q - \bar{Q}$, $L - \bar{L}$ — Magnetic Monopoles?

10^{-34} sec.

Cosmic Inflation?

GRAND UNIFICATION EPOCH

10^{-43} sec.

QUANTUM GRAVITY EPOCH

Cosmic problems

Theory:

- Outcomes of phase transitions (QCD, electroweak, GUTs, ...)
  - Defect formation
  - Baryogenesis

More theory:

- Quantum processes in the presence of horizons
  - Generation of inflationary fluctuations (de Sitter horizon)
  - Hawking radiation (Schwarzschild horizon)

Needs experimental justification...

Coslab: underlying philosophy

Fundamental **processes** that are common to cosmology/particle physics and condensed matter can be tested in the lab.

Suitably designed experiments can simulate cosmology.

The **state** of the system (*e.g.* spectrum of particles, cosmological expansion rate) cannot be justified based on laboratory experiment.

Current focus

Produce analogs of gravitational environment to simulate cosmic inflation, black holes, ...

Overview

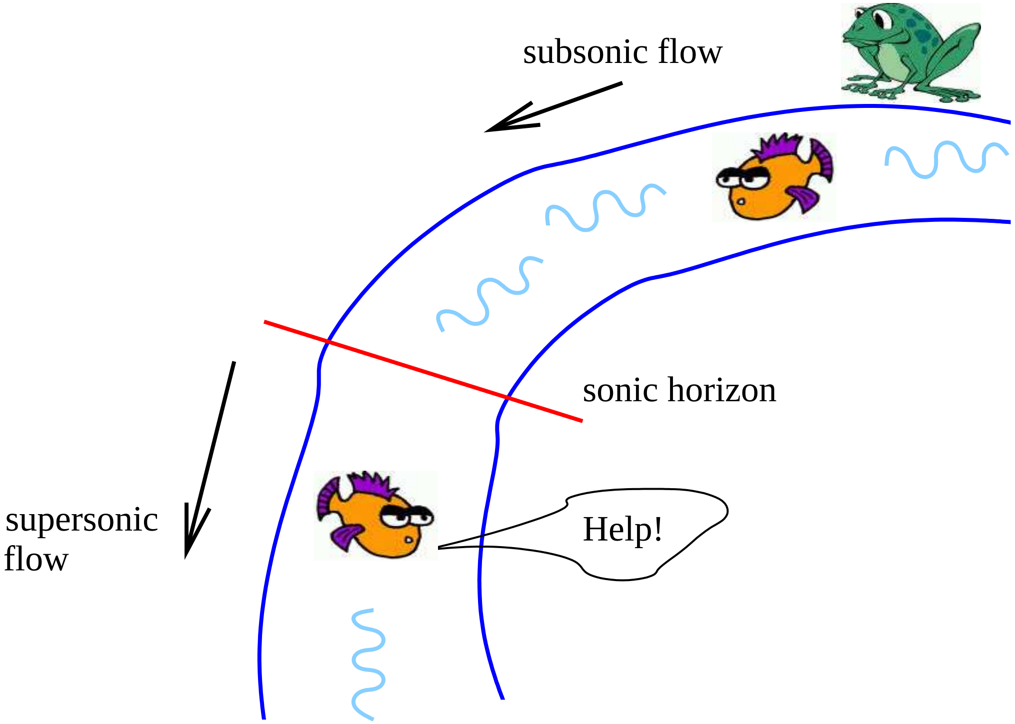
Spontaneous symmetry breaking results in a spatial distribution of an order parameter. Excitations interact with the order parameter and, effectively, propagate on a background metric.

Similary, waves in a fluid effectively propagate on a metric.

Dumbholes

Basic idea: Waterfall

Unruh, 1981



Fluid flow

Irrotational, continuity, Euler equations:

$$\nabla \times \mathbf{v} = 0\,, \quad \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$
$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla p - \nabla \Phi$$

Fluctuations (“sound”): $\rho = \rho_0 + \delta\rho$, $\mathbf{v} = \mathbf{v_0} + \nabla \phi$

Result:

$$\nabla_\mu \nabla^\mu \phi = \frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \phi) = 0$$

The metric

Fluid flow $g_{\mu\nu}$ given by:

$$ds^2 = (c_s^2 - v_0^2)dt^2 + 2v_0 dt dr - dr^2 - r^2 d\Omega^2$$

Painlevé-Gullstrand-Lemaître form of black hole metric[†].

Horizon at $v_0 = c_s$. Sound cannot propagate upstream from horizon.

“Dumbhole” — a (smart) hole that cannot “speak”.

[†] Assuming spherically symmetric, stationary, convergent, background flow.

Hawking radiation

Quantum Field Theory around dumbhole leads to “Hawking sound” just as QED around a black hole gives Hawking light.

Hawking temperature:

$$T_H = \frac{1}{2\pi}\frac{\partial v_0}{\partial r}\bigg|_{hor} = (3 \times 10^{-7}\ K)\left[\frac{c_s}{300\mathrm{m/s}}\right]\left[\frac{1\mathrm{mm}}{R}\right]$$

where $R =$ horizon size

Practical matters

Cold! μK temperatures, tiny power.

Fast! 300 m/s in 1 mm $\sim 10^7 g$

It freezes! High pressure needed to accelerate the flow freezes all known fluids.

It's unstable! Roughness on atomic scales (^{4}He) on container walls lead to instabilities.

It's impossible! Difficulties are insurmountable unless sound speed is very low, in which case the power emitted is too small to be measured.

Unruh, 2001

A generalization and a ray of hope...

Visser, 1998

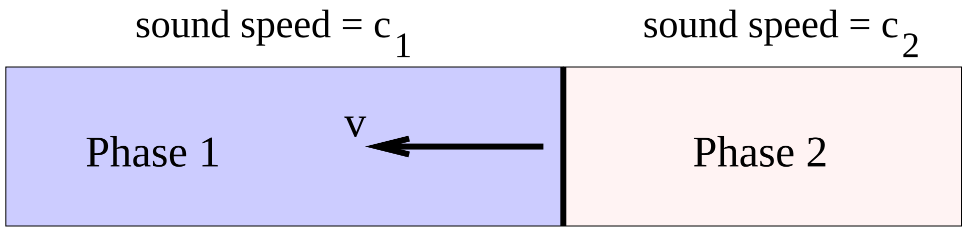
$$T_{sH} = \left(\frac{\hbar}{2\pi k_B} \right) \frac{\partial}{\partial r} (c_s - v) \Big|_{hor}$$

where $c_s = c_s(t, \mathbf{x})$.

Instead of manipulating v, perhaps we can manipulate c_s.

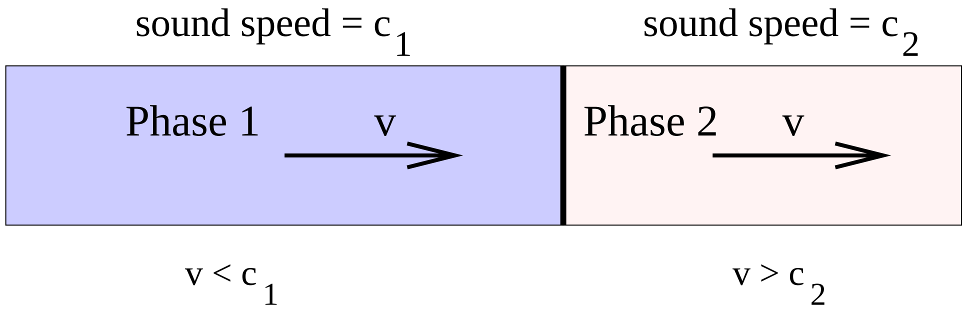
Propagating phase boundaries

TV, gr-qc/0312069, cond-mat/0404480



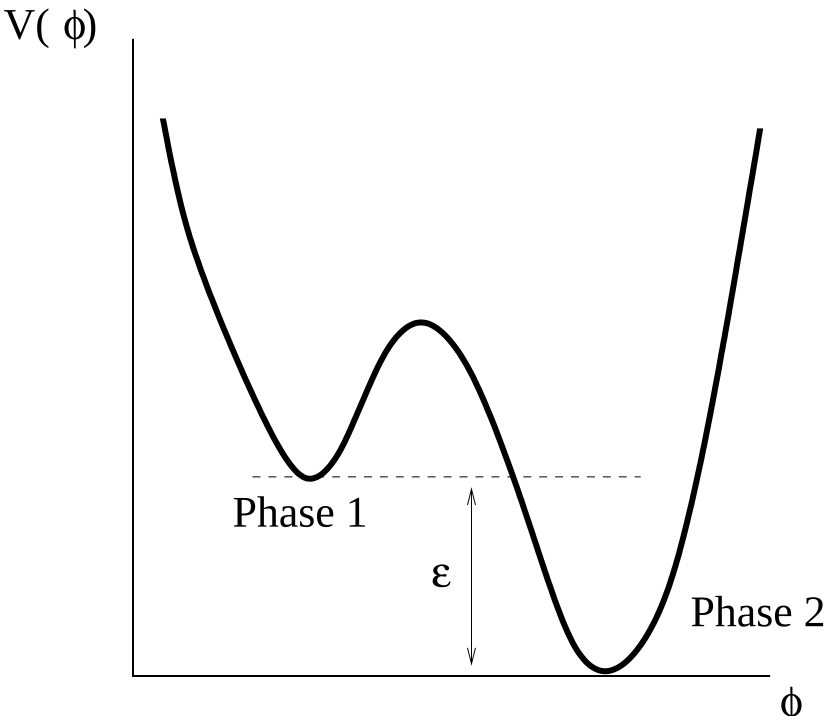
Assume: $c_1 > v > c_2$

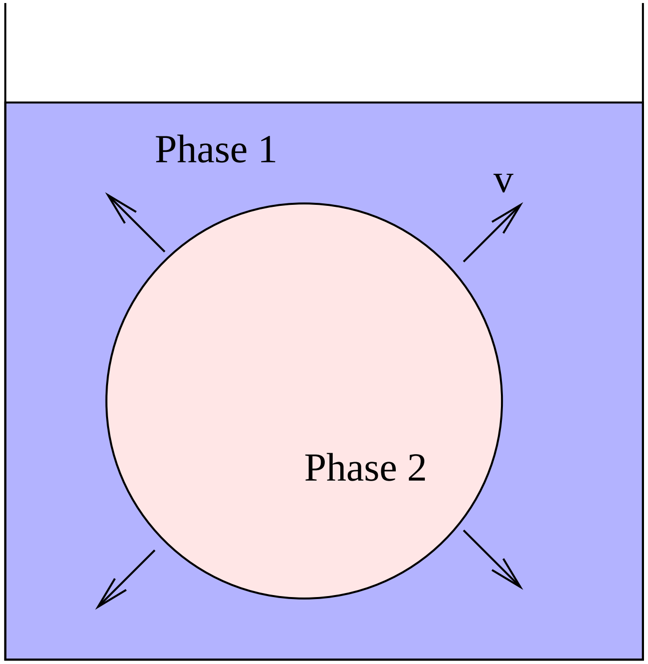
In rest frame of phase boundary:



Phase boundary is a sonic horizon.

First order phase transitions

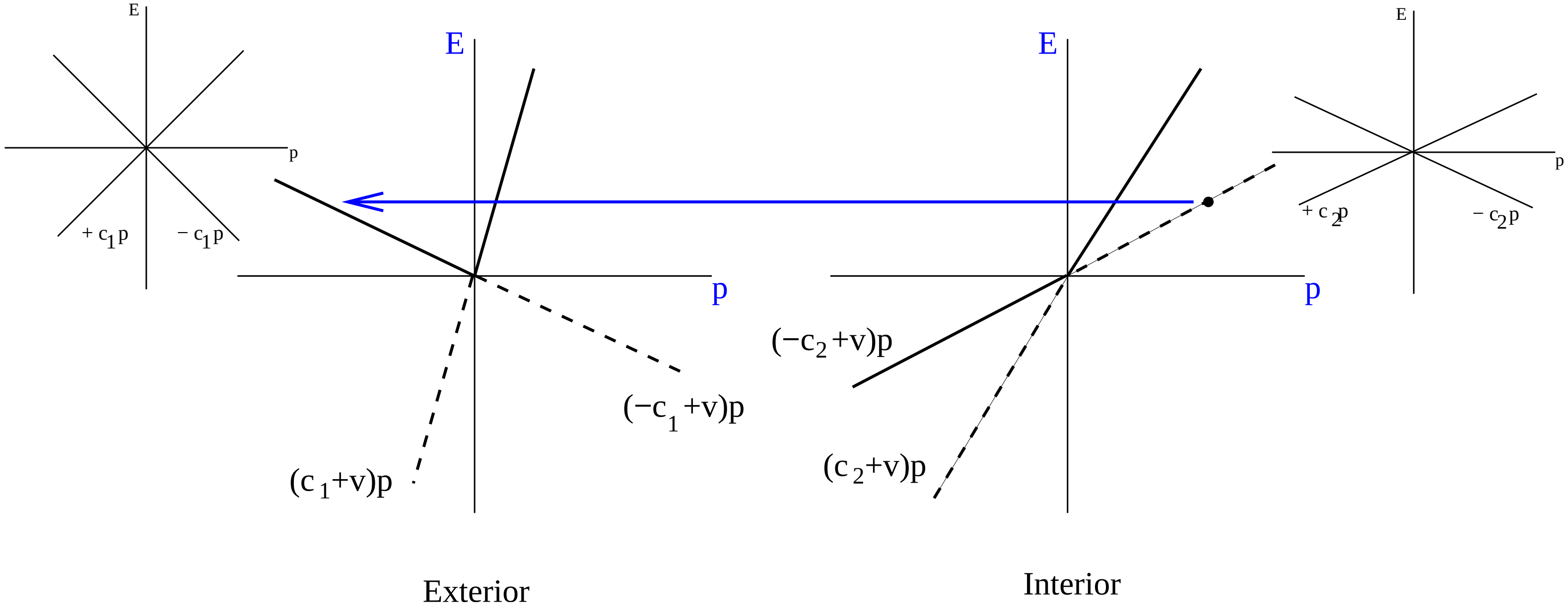




Supercooled system. Decay proceeds by bubble nucleation.
No container walls in 3D.

Hawking temperature: a quick derivation

Volovik, 1999



$$\mathrm{Im}(S) = \mathrm{Im}\int_{-\infty}^{+\infty} p(x)dx = \mathrm{Im}\int_{-\infty}^{+\infty} \frac{E}{v-c(x)}dx = \frac{\pi E}{|c'|}$$

$$\text{Tunneling rate} \propto e^{-2\mathrm{Im}(S)} = e^{-2\pi E/|c'|}$$

The tunneling modes

H. Mathur & TV, in progress

Massless case Schrodinger equation:

$$\left[-ic(X)\partial_x - i\frac{c'(X)}{2}\right]\sigma_z\Psi = i\frac{\partial\Psi}{\partial t}$$

where $X = x - vt$.
Solution: $\Psi^T = e^{-i\omega t}(\psi_1, \psi_2)$

$$\begin{aligned}\psi_1 &= \frac{A_1}{\sqrt{c-v}}\exp\left[i\omega\int\frac{dX}{c-v}\right]\\ \psi_2 &= \frac{A_2}{\sqrt{c+v}}\exp\left[-i\omega\int\frac{dX}{c+v}\right]\end{aligned}$$

Hawking temperature: estimate

$$T_{sH} = \frac{|c'|}{2\pi} = 0.04\ \mathrm{K} \left(\frac{\delta c_s}{300\ \text{m/s}} \right) \left(\frac{100\text{Å}}{\xi} \right)$$

- ξ is the thickness of the phase boundary.
- v determines location at which gradient of c_s is evaluated.
- We will assume that T_{sH} is (roughly) independent of v as long as: $c_1 > v > c_2$.

From theory to lab?

Can this setup be realized experimentally?

Requirements:

- Cold.
- Two adjacent phases.
- High velocity of interface from slow region into fast region.
- Suitable “sound”.

B → A transition in ^{3}He

Cold: $\sim 100\mu$K

Two adjacent phases: A and B

High velocity of interface: v_{BA} (*i.e.* growing bubble of B in A) observed to be as high as 67 cm/s. Buchanan et. al., 1986

$v_{AB} \sim$ several cm/s when driven by magnetic fields.

Bartkowiak et. al., 2000

Excitations: Many collective excitations are present. In particular, some fermionic quasiparticles have: $c_{A\perp} = 3$ cm/s, $c_B = 55$ m/s.

Quasiparticles in AB phases

$$\mathrm{A-dispersion:}\quad E(\boldsymbol{p}) = \pm\left[v_F^2(|\boldsymbol{p}|-p_F)^2 + \frac{\Delta_A^2}{p_F^2}(\hat{\boldsymbol{l}}\times\boldsymbol{p})^2\right]^{1/2}$$

$$\mathrm{B-dispersion:}\quad E(\boldsymbol{p}) = \pm\left[\{\epsilon(\boldsymbol{p})-\mu\}^2 + \Delta_B^2\frac{\boldsymbol{p}^2}{p_F^2}\right]^{1/2}$$

Near the node in the A-phase, $\boldsymbol{p} = \pm p_F\hat{\boldsymbol{l}} + \delta\boldsymbol{p}$ with $\delta\boldsymbol{p}\cdot\hat{\boldsymbol{l}} = 0$:

$$\mathrm{A:}\quad E = \pm c_A|\delta\boldsymbol{p}|$$

$$\mathrm{B:}\quad E(\boldsymbol{p}) = \pm\left[v_F^2(\delta p)^2 + \Delta_B^2\right]^{1/2},\quad \delta p \equiv p - p_F$$

Issues with the AB system

Massless (gapless) quasiparticles on A-side are massive (gapped) on B-side. Full dispersion relations are not of relativistic form.

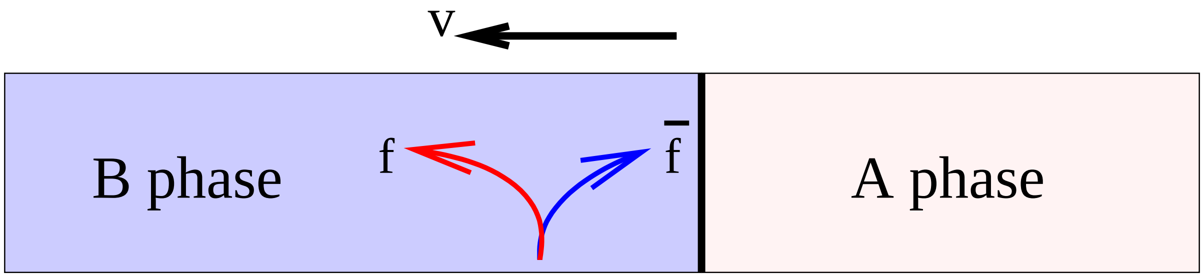
Therefore the straightforward metric analogy breaks down.

However the black hole analogy need not break down!

There is still a sonic horizon in which quasiparticles can fall in but from which they can't emerge†. Hawking radiation should still be emitted!

†Even a static AB interface is a reflecting mirror for quasiparticles impinging from the A phase side. But this is only an energy barrier preventing escape (not a "velocity barrier") and does not lead to Hawking radiation.

Hawking radiation without a metric

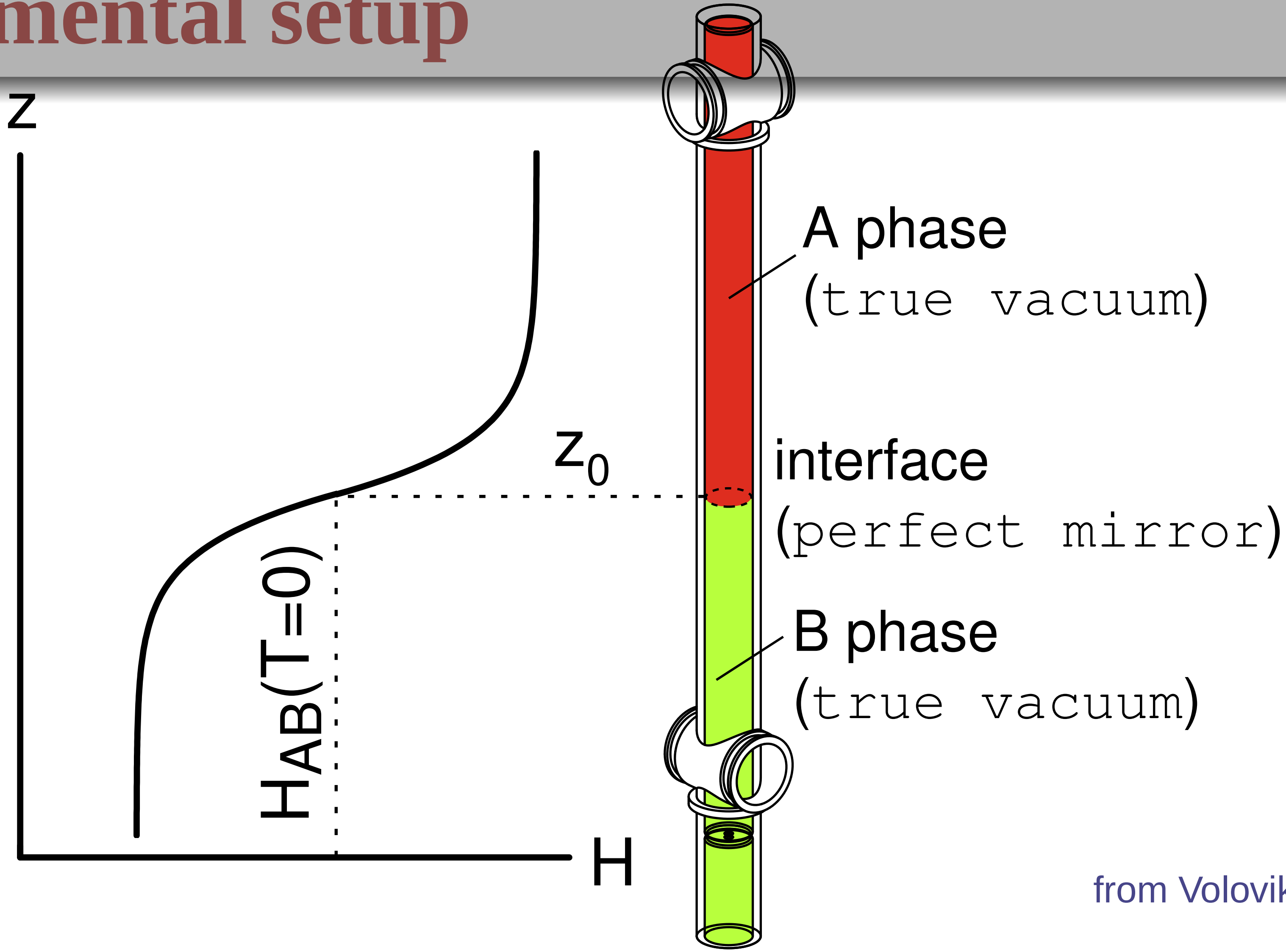


Vacuum fluctuations in the B phase produce an $f\bar{f}$ pair, say with f having positive energy and $\bar{f}$ having negative energy. $\bar{f}$ falls into the dumbhole, never to be seen again. f escapes, and forms Hawking radiation.

Gibbons, 1979

Can this radiation be seen?

Experimental setup



from Volovik’s book

FIG. 29.1. Interface between two vacua stabilized by the vacuum pressure induced by external magnetic field. Interface is at $z = z_0$, where $H(z_0) = H_{AB}$. It separates true vacuum of ^{3}He-A, from the true vacuum of ^{3}He-B.

Oscillations of the He-3 AB interface

Bartkowiak et al (Lancaster), 2000

Drive interface using: $B(t,z) = B_0(z) + B_{AC}\ \sin(2\pi\nu t)$

Measure energy flux in quasiparticles in asymptotic region.

Equation of motion: $-k(x - x_0(t)) - \gamma\frac{dx}{dt} = 0$

where $x_0(t) = a\sin(\omega t)$ is the position if $\gamma = 0$. Then,

$$v(t) = v_0\cos(\omega t - \phi) + O(e^{-\kappa t})$$

$$\kappa = \frac{k}{\gamma}\ ,\quad v_0 = \frac{a\kappa\omega}{\sqrt{\kappa^2 + \omega^2}}\ ,\quad \tan\phi = \frac{\omega}{\kappa}$$

Therefore $v_0 \propto \omega$ for $\omega << \kappa$ and v_0 is independent of ω for $\omega >> \kappa$.

Radiation: $P = \gamma v^2$ (force×velocity)

The data

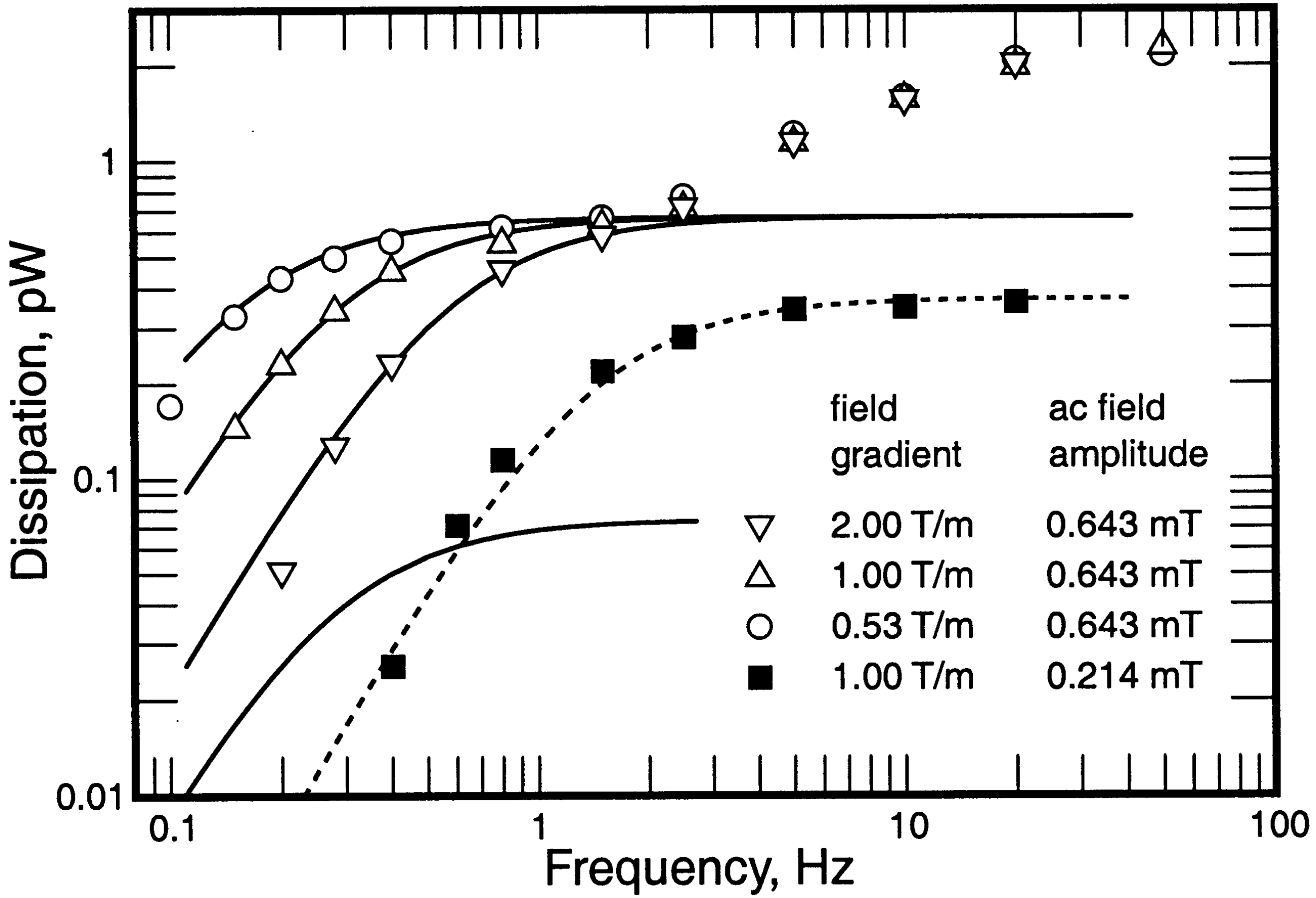


Fig. 2. The dissipation of an oscillating A–B boundary as a function of its frequency, see text.

Understanding the data

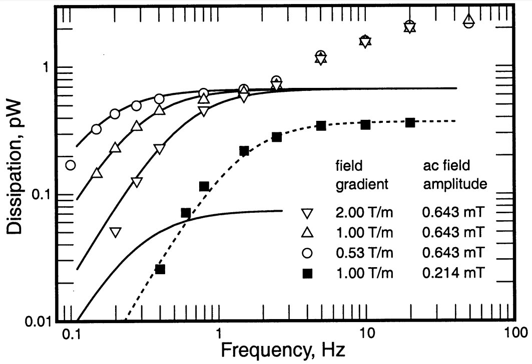


Fig. 2. The dissipation of an oscillating A–B boundary as a function of its frequency, see text.

Three issues:

1. Theoretical γ small by several orders of magnitude.

   Leggett & Yip, 1990

2. For the smaller B_{AC}, the required γ is about 5 times smaller (see lower curve).

3. Data clearly shows an unexpected increase in dissipation at frequencies $\nu > 1$ Hz when $B_{AC} = 0.643$ mT. Points to a new source of radiation.

Can Hawking radiation be playing a role at high frequencies?

Interface motion at high driving frequencies

At $\nu \gg 1$ Hz:

- Interface probably still oscillates at frequency $\sim \omega$.
- Data indicates that v departs from the low frequency behavior. Assume that the amplitude of v grows with ω.

For *illustrative* purposes take

$$v = \alpha\omega \cos(\omega t - \tilde{\phi})$$

where the amplitude α is an ω independent parameter in the frequency range of interest.

The exact form of v is not crucial for us. Even the power of ω in the amplitude could be different from 1.

Dissipation due to Hawking radiation

Hawking radiation occurs only when $v = \dot{x} > c_A$.

- *Assume* thermal Hawking radiation at T_{sH}.

For j "light" species of radiation, this gives[†]:

$$P_H = j\sigma_s T_{sH}^4 A \left(\frac{\delta t}{\tau}\right) = j\sigma_s T_{sH}^4 A \frac{1}{\pi} \cos^{-1}\left(\frac{c_A}{2\pi\alpha\nu}\right)$$

Write this as

$$P_H = \frac{P_0}{\pi} \cos^{-1}\left(\frac{\nu_*}{\nu}\right)$$

[†] $T_{sH} \approx$ 3mK $> \Delta_B \approx$ 1.7mK

Comparison with data

$$P_0 \approx 116 \left(\frac{j}{2}\right) \left(\frac{d}{4.3\text{mm}}\right)^2 \left(\frac{\delta c_s}{60\text{m/s}}\right)^2 \left(\frac{100\text{Å}}{\xi}\right)^4 \text{p}W$$

where d is the cell diameter (4.3 mm in the experiment). This is in the right range if $\xi \sim 300$ Å.

The ν dependence ($\cos^{-1}(\nu_*/\nu)$) qualitatively agrees with data if ν_* is treated as a free parameter.

How about the value of the critical frequency ν_*?

Comparison contd.

$$\nu_* = \frac{c_A}{2\pi\alpha}$$

If $\alpha = B_{AC}/\nabla B_0|_{\text{interface}}$ then

∇B_0 T/m	δB mT	ν_* Hz
2.00	0.643	14.8
1.00	0.643	7.4
0.53	0.643	3.9
1.00	0.214	22.2

But no reason to adopt the low frequency $\alpha = B_{AC}/\nabla B_0$.
Explanation can only work if α is independent of ∇B_0.

Conclusions

- Experiments on phase transitions have successfully been done to test ideas that are relevant to cosmology.
- Condensed matter experiment offers a hope to study quantum field theory in curved spacetime.
- Experiments in superfluid ^{3}He may be suitable for studying Hawking radiation.
- Condensed matter experiments may be able to shed light on other cosmic problems.